This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

EducT
149
16.336

KEY
TO
PLANE GEOMETRY

DURELL - ARNOLD

Educ. T 144.18.325

3 2044 097 046 536

Educ T 144.44.325

3 2044 097 046 536

KEY

TO

PLANE GEOMETRY

BY

FLETCHER DURELL, Ph.D.

HEAD OF THE MATHEMATICAL DEPARTMENT IN THE
LAWRENCEVILLE SCHOOL

AND

E. E. ARNOLD

SPECIALIST IN MATHEMATICS IN THE UNIVERSITY OF
THE STATE OF NEW YORK

CHARLES E. MERRILL COMPANY
NEW YORK AND CHICAGO

Educ T 149.16.325

HARVARD
UNIVERSITY
LIBRARY

Copyright, 1916,
By CHARLES E. MERRILL CO.

KEY TO PLANE GEOMETRY

PAGE 11

17. He first sets out two of the trees in position and then sights through them and places the other trees in line with the two already set out.

22. Straight, *I*; curved, *C, O, Q, S, U*; broken, *A, E, F, H, K, L, M, N, T, V, W, X, Y, Z*; mixed, *B, D, G, J, P, R*.

PAGE 18

3. 47°; 137°.

4. 32° 41′; 27° 36′ 17″; 122° 41′; 117° 36′ 17″.

5. Construct a right angle having the same vertex as the given angle and a side in common with this angle, and including the given angle as a part of itself.
To construct the supplement, produce a side of the given angle through the vertex.

6. Obtuse.

7. 6°.

8. 60°; 90°; 150°.

9. 135°; 22° 30′; 7° 30′.

10. 10 min.; $8\frac{1}{3}$ min.; 40 min.; 2 hr.; 1 hr. 40 min.; 8 hr.

11. 810°.

PAGE 19

12. $\frac{2}{3}$.

13. 30°; 90°; $22\frac{1}{2}$°; 15°; $1\frac{7}{8}$°.

14. 1 lb.; $\frac{2}{3}$ lb.; $1\frac{1}{2}$ lb.; $2\frac{1}{3}$ lb.; $\frac{1}{10}$ lb.; 6 lb.

15. Acute; obtuse; right.

18. Obtuse; acute.

19. Reflex; acute.

20. Right; obtuse; acute.

21. Right angle, for, denote the given angle by x. Then $(180° - x) - (90° - x) = 90°$.

22. (*a*) $r = t$. (*b*) $r = t$. (*c*) $t > r$, for s and r are acute. $\therefore t$ is obtuse. (*d*) $r > t$, for s is acute and $\therefore r$ is obtuse.

23. (*a*) $m > p$. (*b*) $m = p$. (*c*) $m < p$. (*d*) Denote the comp. of m by x. Then $x + m = 90°$, $x + p = 180°$. $\therefore p - m = 90°$. $\therefore p = m + 90°$. $\therefore p > m$.

PAGE 20

24. $x = 2\,(90° - x)$. $\therefore\ x = 60°$. *Ans.*

25. $x = \frac{1}{3}\,(180° - x)$. $\therefore\ x = 45°$. *Ans.*

26. $x = \frac{4}{5}\,(180° - x)$. $\therefore\ x = 80°$. *Ans.*

27. (1) $x = 90° - x + 12°$. $\therefore\ x = 51°$. *Ans.*
(2) $x = 180° - x + 15\frac{1}{2}°$. $\therefore\ x = 97°\,45'$. *Ans.*

28. $90° - x + 180° - x = 126°$. $\therefore\ x = 72°$. *Ans.*

29. $180° - x = 4(90° - x)$. $\therefore\ x = 60°$. *Ans.*

PAGE 21

2. Yes, by placing the edge of the ruler on the pipe in a direction parallel to the length of the pipe. No.

3. No.

4. Place the straight edge on the surface in various directions. In all positions every point of the straight edge should be in contact with the surface of the tennis court.

5. Yes.

PAGE 24

1. Ax. 7.

2. 140°; 210°; Ax. 7.

3.

$$\begin{array}{rcr} 7 & = & 7 \\ -2 & = & -2 \\ \hline 5 & = & 5 \end{array}$$

4. (1)

$$\begin{array}{rr} 5 = & 5 \\ \times 2 & \times 2 \\ \hline 10 = & 10 \end{array}$$

(2) $12 = 12$. Dividing each of these by 3, $4 = 4$.

5. $8 = 8$. $\therefore\ \sqrt[3]{8} = \sqrt[3]{8}$, or $2 = 2$.

6. Ax. 4.

PAGE 25

7. Ax. 2.

8. Use the diagram of Ex. 7. Thus, if $LN = MO$, then $LM = NO$ (Ax. 3).

11. Ax. 1. **12.** Ax. 2. **13.** Ax. 9.

14. Through two points only one straight line can be passed; and a right angle is half of a straight angle.

PAGE 28

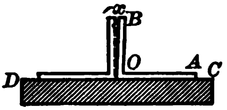
1. Three.
2. Six.
3. In Ex. 1 each point helps to locate two lines; in Ex. 2, each point helps to locate three lines. Hence, a point in the latter case does $1\frac{1}{2}$ times as much work, or is $1\frac{1}{2}$ times as efficient as in the former case.

5. 140.° 6. $r = 43°$, etc.

8. $\angle ABP = 40°$; $\angle PBQ = 50°$; $\angle QBC = 40°$.

9. $\angle COD = 180° - 142° = 38°$. $\angle COB = 120° - 38° = 82°$. *Ans.*

10.



Place AO, one of the inside edges of the square, in contact with the straight edge CD. Then repeat the act, placing O at the same point on CD, with OA pointed in the opposite direction from the first position.

By § 23, the sum of two right angles must be a straight angle.

PAGE 30

2. Any two of the angles which are not vertical.
3. 157° (§ 69).
4. $s = p = 153° - 107° = 46°$. *Ans.*
5. $t = q = 138°$ (see Ex. 9, p. 28).
6. $4r = 180° - 24°$. $\therefore r = x = 39°$. *Ans.*

PAGE 32

2. 72°; 104°.
3. Add the complement to 90°; subtract 90° from the supplement.
4. 150°; 12° 30′.
5. As the sides of a triangle; two || lines, one through each point, and a third line through the two given points; etc.

6. 12 in. 7. No. 8. No.

10. Denote the angle of the square by y. Then
$2(y + e) = 2y + x$. $\therefore 2e = x$. $\therefore e = \frac{1}{2}x$.

PAGE 33

1. $EF = 18$; $\angle E = 70°$; $\angle F = 50°$.

PAGE 35

1. $DE = 24$; $EF = 27$; $\angle E = 55°$.
1. (Group 9). Use § 79.
2. § 79. 3. § 79. 4. § 79.
5. Prove $\triangle DCF = \triangle ACB$ by § 79. 217 yd.
6. § 80.

PAGE 36

7. § 80. 8. § 80.
9. $\angle ABO = \angle CBO$ by § 66. Use § 80.
10. Use § 80. 137 yd.

PAGE 37

1. By the principle proved in § 82, $\angle C = \angle A = 67°$.

PAGE 39

1. $\angle BAC = \angle BCA$ (§ 82). $\therefore p = r$ (§ 66).
   Then prove $\triangle DAB = \triangle BCF$ by § 79.
2. Use § 82 twice and Ax. 3.
3. § 83. 4. § 83.
5. Draw AC and use § 83.

PAGE 41

2. Construct a right angle and bisect it. Also through the vertex draw a line $\perp$ the bisector.

PAGE 50

2. No, for $d = 110°$, and hence d and f are not supplementary. Use § 99.
1. (Group 11). $\angle MOQ = \angle POL$ (§ 69). $PO = OQ$ (Hyp.).
   $\angle MQO = \angle OPL$ (§ 96). $\therefore \triangle POL = \triangle OMQ$ (§ 80).
2. $\angle A = \angle C$ (§ 82); $\angle D = \angle C$ (§ 96), etc.
3. Prove $\triangle ABP = \triangle PCD$ by § 80, etc.
4. $x = p$ (§ 69); $y = q$ (§ 69); $x = y$ (Ax. 1). $\therefore AB \parallel CD$ (§ 89).
5. $\angle ABC = \angle BCD$ (Ax. 2), etc.

PAGE 51

6. $y = m$ (§ 82). $x = y$ (§ 97); $l = m$ (§ 97), etc.
7. Prove $\triangle ABD = \triangle BFC$ by § 79, etc.
8. $\angle B = \angle C$ (§ 82). Then use § 96 twice and Ax. 1.
9. $\angle CBE = 115°$ (Ax. 7). $\therefore \angle ABE = 65°$ (§ 32).
$\therefore BE \parallel CD$ (§ 91).
10. $\angle B = \angle i$ (step 6, p. 43), etc.

PAGE 53

1. 62°.
2. 53° 45′.
3. 60°.
4. 45°.
5. No.

PAGE 54

6. 138°.
7. 142°; 115°; 103°.
8. 71°.
9. 80°.
10. 78°; 78°; 24°.
11. Construct an equilateral triangle and bisect one of its angles. Bisect an angle of 30°.
12. Construct the supplement of 60°. $75° = 45° + 30°$.
13. $150° = 90° + 60°$. $195° = 180° + 15°$.
15. Construct an equilateral triangle and a perpendicular to the base through an extremity of the base.
16. See Ex. 12.
17. Construct an angle of 45° at each end of the 2-in. line.
19. Corr. $\measuredangle$ are = (§ 107). $\triangle$ are not equal.
20. Through the vertex of the acute angle construct a $\perp$ to one side of the angle.
21. Produce one side of the angle through the vertex.

PAGE 57

1. Use § 114.
2. § 114.
3. $\angle B$ (§ 114).
4. § 114.
5. $\angle OAQ = \angle OBP$ (§ 114).
$\angle AOQ = \angle BOP$ (§ 69).
$\angle AOB = \angle POQ$ (§ 69).
$\angle AQO = \angle OQC = \angle APC = \angle APB$ (§ 63).
6. See Ex. 1, p. 39.

PAGE 60

1. Use § 117. $PQ = PR$ (corr. sides of = △).

PAGE 61

1. Use § 110.

PAGE 63

4. The locus is a straight line, parallel to the top of the level track and 1 ft. above it.

5. The locus is a straight line ⊥ the line joining the two given points at the midpoint of this line.

6. Last sentence of § 123.

PAGE 69

1. Ineq. Ax. 2.

2. (1) (2) (3)

$$\begin{array}{r} 7 > 5 \\ +3 + 3 \\ \hline 10 > 8 \end{array} \qquad \begin{array}{r} 7 > 5 \\ -2 - 2 \\ \hline 5 > 3 \end{array} \qquad \begin{array}{r} 7 > 5 \\ 5 > 3 \\ \hline 12 > 8 \end{array}$$

3. (1) See Ex. 1. (2) $10 > 8$. $\therefore 5 > 4$ Ill. of Ineq. Ax. 3,

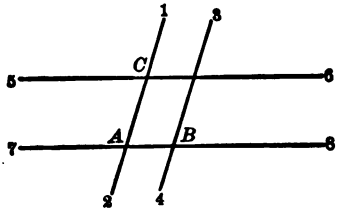
$$\begin{array}{rcr} 10 & = & 10 \\ -6 & > & -2 \\ \hline 4 & < & 8 \end{array}$$

4. $9 > 7 > 3$. $\therefore 9 > 3$.

5. § 134.

PAGE 72

1. Four; five. Three points will not determine a pair of parallel lines, for two pairs might be passed through the three points; thus through the points A, B, C the ∥ lines 12 and 34 might be passed, also the parallel lines 56 and 78.



2. No.

PAGE 74

1. Hence, $\angle APB$ is obtuse. Use § 138.

2. $AB + BF > AF$ (§ 78). Also $FC + CD > FD$ (§ 78). $AD = AD$ (Ident.). Adding by Ineq. Ax. 1, $AB + BC + CD + DA > AF + FD + AD$.

3. $BD + DF > DF$ (§ 78). To each of these unequals add $AD + FC$ (Ineq. Ax. 1).
4. Use $\triangle$s ABD and ADC, and § 138.
5. Use $\triangle$s ABD and DBC, and § 139.

PAGE 76

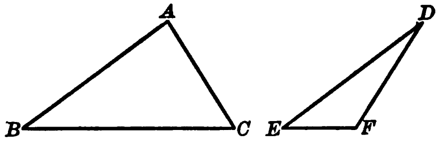
2. Yes; no.
3. $\angle$s of a square are rt. $\angle$s; $\angle$s of a rhombus are oblique. Have same number of sides; sides are equal; opposite sides are parallel.
4. 48 in.
5. Rhombus, rhomboid, parallelogram, quadrilateral.
6. Have same number of sides; opposite sides are parallel.
7. Two $\triangle$s; isosceles.
9. Construct a rectangle and a rhomboid whose corresponding sides are equal.

PAGE 77

1. $\angle r = \angle s$ and $\angle q = \angle p$ (corr. $\angle$s of $=$ $\triangle$s).
   $\therefore \angle BAD = \angle BCD$ (Ax. 2).
2. Produce BC to F and DC to H.
   Then $\angle BAD = \angle HCF$ (§ 112), $\angle BCD = \angle HCF$ (§ 69).
   $\therefore \angle BAD = \angle BCD$ (Ax. 1).

PAGE 78

1. Four, viz.: AFD and BFC; AFB and DFC; ABD and CBD; ABC and ADC.
2. $180° - p° - q°$.
3. $90° - 2x°$.
7.



Construct the triangles ABC and DEF in which $\angle B = \angle E$, $AB = DE$, and $AC = DF$. See text-book, p. 159.

PAGE 79

1. Geom. Ax. 2, Post. 1 (§ 46), § 82 (twice), Ax. 2 (§ 41), Ax. 7 (§ 41), § 79.

PAGE 80

1. Use § 98. 2. 43°; 137°.

3. Denote the given angles by x and $3x$.
Then $x + 3x = 180°$ (§ 98). $\therefore$ $x = 45°$; $3x = 135°$. *Ans.*

4. $a°$; $180° - a°$.

5. AC, longest; AB, shortest (§ 135).

PAGE 81

1. § 159.

2. No. (1) Converse of § 69, viz.: if two angles are equal, they are vertical. (2) Converse of § 112, viz.: if two angles are equal, their corresponding sides are parallel.

3. If we know that the diagonals of a quadrilateral bisect each other, by § 162, we at once know without effort that the given quadrilateral is a parallelogram.

PAGE 82

1. $AD \parallel BC$ (§ 92). Then use § 161. AB is 312 yd. (§ 155).

2. $BH = CF$ (Ax. 2). Then prove $\triangle ABH = \triangle DFC$ by § 79.

3. In $\triangle DPC$ and AQB, $DC = AB$ (§ 155), $DP = BQ$ (Hyp.), $\angle PDC = \angle QBA$ (§ 96). Use § 79.

4. Prove $\triangle RAB = \triangle DCS$ by § 79. Also $\triangle RAD = \triangle BCS$ by § 79, etc. Use § 160.

5. In $\triangle$ AOP and QOC, $AO = OC$ (§ 159), $\angle PAO = \angle QCO$ (§ 96), $\angle AOP = \angle QOC$ (§ 69), etc.

PAGE 83

8. Prove both AD and HF $\parallel$ and $= BC$ by §§ 146, 101, 155, and Ax. 1.

9. Use figure, p. 81 (text-book). Then $AB + BC > AC$ (§ 78). $BC + CD > BD$ (§ 78). $CD + AD > AC$ (§ 78). $AD + AB > BD$ (§ 78). Add, and divide by 2 (Ineq. Axs. 1 and 2).

12. Construct $\angle$ of 45° at each end of the given diagonal. Use §§ 102, 115, 92.

13. Construct the $\perp$ bisector of one diagonal (§ 128), and on it from the point of intersection mark off parts $= \frac{1}{2}$ the other diagonal.

PAGE 84

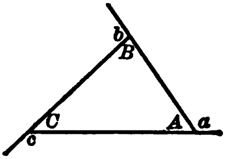
2. No. All polygons except triangles.

PAGE 86

1. 4.
2. 8.
3. 28.
4. 36.
5. 540°.
6. 1080°.
7. 1440°.
8. 2880°.
9. 120°.
10. $128\frac{4}{7}$°.
11. 144°.
12. $157\frac{1}{2}$°.
13. 181°.
14. 118°.

16. Denote the angles of the quadrilateral by a, b, c, d. Let $a + b = 180°$. Then $a + b + c + d = 360°$ (§ 167). $\therefore\ c + d = 180°$ (Ax. 3).

17. Complete the figure thus:



Then
$$A + a = 180° \text{ (§ 64)}$$
$$B + b = 180° \text{ (§ 64)}$$
$$C + c = 180° \text{ (§ 64)}$$

$\therefore\ A + B + C + a + b + c = 540°$. (Ax. 2).

But $A + B + C = 180°$ (§ 102). $\therefore\ a + b + c = 360°$ (Ax. 3).

18. Denote the angles of the ▱ in order by a, b, c, d. If $a = 90°$, then $b = 90°$ (§ 98), $c = d = 90°$ (§ 155).

19. $\left(\dfrac{2n-4}{n}\right) 90 = 108$, whence $180n - 360 = 108n$; $72n = 360$. $\therefore\ n = 5$. *Ans.*

20. No; yes; no.

PAGE 89

6. Divide the given perimeter into 3 equal parts by § 170.

1. (Group 16). Use § 80.

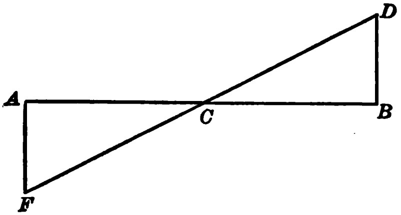
2. Use figure, p. 64 (text-book). Thus,
Given ∠ ABC, $BQ = BR$, BP the bisector of ∠ ABC.
To prove △ BQP = △ BRP.
Proof. Use § 79.

3. Use § 79. Use figure of Ex. 2, p. 92.

4. § 83. Also △ BAE = △ EDC. For ∠ BAC = ∠ DCA (corr. ∡ of = ▲). ∠ DAC = ∠ BCA (same reason). ∴ ∠ BAE = ∠ DCE (Ax. 3). $AB = DC$ (Hyp.). ∠ B = ∠D (corr. ∡ of = ▲). ∴ △ BAE = △ DEC (§ 80).

5. Use § 80.

6.

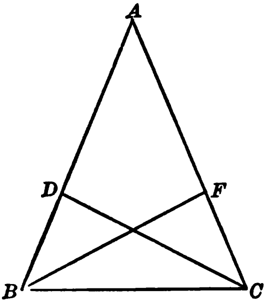


Given ACB and FCD straight lines; $AC = CB$; AF and $BD \perp AB$.
To prove $\triangle ACF = \triangle CDB$.
Proof. $AC = CB$ (Hyp.); $\angle ACF = \angle DCB$ (§ 69); $\angle A = \angle B$ (§ 63). $\therefore$ $\triangle ACF = \angle CDB$ (§ 80).

7. Use figure, p. 78, (text-book). Thus,
Given line AC intersecting the line BD at the point F, and $AF = FC$, $BF = FD$.
To prove $\triangle BFC = \triangle AFD$, and $\triangle BFA = \triangle CFD$.
Proof. Use § 79.

PAGE 90

8.



Given $\triangle ABC$, $AB = AC$, and $BD = FC$.
To prove $\triangle DBC = \triangle FBC$, and $\triangle ADC = \triangle AFB$.
Proof. In ▲ DBC and FBC, $BC = BC$ (Ident.), $DB = FC$ (Hyp.), $\angle DBC = \angle FCB$ (§ 82). $\therefore$ $\triangle DBC = \triangle FBC$ (§ 79). Also in ▲ ABF and ADC, $\angle A = \angle A$ (Ident.), $AB = AC$ (Hyp.), $DB = FC$ (Hyp.). $\therefore$ $AD = AF$ (Ax. 3). $\therefore$ $\triangle ABF = \triangle ADC$ (§ 79).

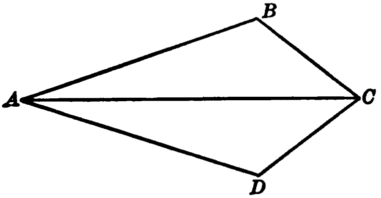
9. $AC = BD$ (Ax. 2). Use § 83.

10. The angles included by the legs are = (§ 63). $\therefore$ The ▲ are = (§ 79).

11. $AP = QC$ (Ax. 3). Use § 79.

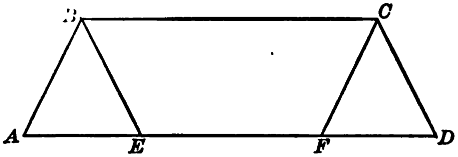
12. Two pairs. Thus, using the figure of Ex. 8,
Given the $\triangle ABC$, $AB = AC$, $CD \perp AB$, $BF \perp AC$.
To prove $\triangle DBC = \triangle BFC$, and $\triangle ABF = \triangle ADC$.
Proof. In $\triangle$ DBC and BFC, $BC = BC$ (Ident.), $\angle DBC = \angle FCB$ (§ 82). $\therefore$ $\triangle DBC = \triangle BFC$ (§ 110). Also in $\triangle$ ADC and ABF, $AB = AC$ (Hyp.), $\angle A = \angle A$ (Ident.). $\therefore$ $\triangle ADC = \triangle ABF$ (§ 110).

13.



Given the quadrilateral $ABCD$, $AB = AD$, and $\angle BAC = \angle DAC$.
To prove $\triangle BAC = \triangle DAC$.
Proof. Use § 79.

14.



Given $BC \parallel AD$, $AB = CD$. $BE \parallel CD$, $CF \parallel AB$.
To prove $\triangle ABE = \triangle CFD$.
Proof. In $\triangle$ ABE and CFD, $AB = CF$, $BE = CD$ (§ 155). $\angle ABE = \angle FCD$ (§ 112). $\therefore$ $\triangle ABE = \triangle FCD$ (§ 79).

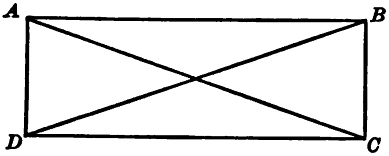
PAGE 92

1. See Ex. 1, p. 89.
2. See Ex. 3, p. 89.
3. See Ex. 5, p. 89.
4. See Ex. 8, p. 90.

PAGE 93

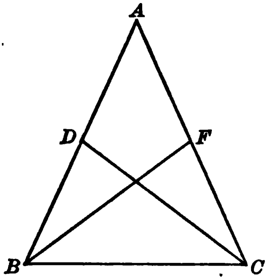
5. See Ex. 1, p. 50.
6. Use § 79.
7. See Ex. 12, p. 90.

8.



Given the rectangle $ABCD$.
To prove the diagonals AC and BD equal.
Proof. In the $\triangle$s ADC and BDC, $DC = DC$ (Ident.), $AD = BC$ (§ 155), $\angle ADC = \angle BCD$ (§§ 149, 63). $\therefore \triangle ADC = \triangle DBC$ (§ 79). $\therefore AC = DB$ (corr. sides of = $\triangle$s).

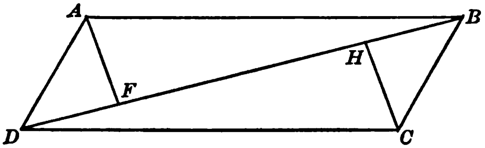
9.



Given $\triangle ABC$, $AB = AC$, D the midpoint of AB, and F the midpoint of AC.
To prove $BF = DC$.
Proof. In the $\triangle$s ABF and ADC, $AB = AC$ (Hyp.). $AF = AD$ (Ax. 5.) $\angle A = \angle A$ (Ident.). $\therefore \triangle ABF = \triangle ADC$ (§ 79). $\therefore BF = DC$.

10. Use the figure of Ex. 8, p. 90.
Given the $\triangle ABC$, $CD \perp AB$, and $BF \perp AC$; and $CD = BF$.
To prove $AB = AC$.
Proof. In the $\triangle$s CDB and BFC, $BC = BC$ (Ident.), $DC = FB$ (Hyp.). $\therefore \triangle DBC = \triangle FBC$ (§ 117). $\therefore \angle DBC = \angle FCB$ (corr. $\angle$s of = $\triangle$s). $\therefore AB = AC$ (§ 115).

11. Given $ABCD$ a $\square$, BD a diagonal, AF and $CH \perp DB$.



To prove $AF = HC$.

Proof. In the ▲ AFB and DHC, $AB = DC$ (§ 155), $\angle ABF = \angle CDH$ (§ 96). $\therefore \triangle AFB = \triangle DCB$ (§ 110), etc.

12. Use § 79.

13. Use §§ 82, 79.

14. Put the letter B at the vertex. Then $\angle BAP = \angle BPA$ (§ 82). $\therefore \angle OAP = \angle OPA$ (Ax. 5). $\therefore OA = OP$ (§ 115).

15. $\angle B = \angle C$ (§ 82). $\angle ADE = \angle B$ (§ 97). $\angle AED = \angle C$ (§ 97). $\therefore \angle AED = \angle ADE$ (Ax. 1). $\therefore AD = AE$ (§ 115).

16. $\triangle BAC = \triangle DAC$ (Ex. 4, p. 89). $\therefore \angle DAC = \angle BCA$ (corr. ∡ of = ▲). $\therefore AE = EC$ (§ 115).

17. Use ▲ ADQ and PBC and § 79.

PAGE 94

1. See Ex. 1, p. 89.

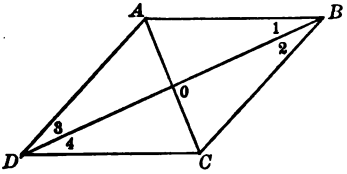
2. See Ex. 5, p. 89.

3. See Ex. 4, p. 89.

4. See Ex. 8. p. 90.

5. See Ex. 12, p. 90.

6.



Given $ABCD$ a ▱ in which $AB = BC = CD = DA$.

To prove $\angle ADB = \angle CDB$, and $\angle ABD = \angle CBD$; also $\angle DAC = \angle BAC$, and $\angle DCA = \angle BCA$.

Proof. $\angle 1 = \angle 4$ (§ 96). But in $\triangle ABD$, $AB = AD$ (Hyp.). $\therefore \angle 1 = \angle 3$ (§ 82). $\therefore \angle 3 = \angle 4$ (Ax. 1), etc. Or, the theorem may be proved by the use of ▲.

7. See Ex. 1, p. 39.

8. See Ex. 15, p. 93.

PAGE 95

9. $\angle DCE = \angle ECA$ (Hyp.). $\angle A = \angle ECA$ (§ 79). $\angle B = \angle DCE$ (§ 97). $\angle A = \angle B$ (Ax. 1).

10. $\angle DCE = \angle B$ (§ 97). $\angle A = \angle ECA$ (§ 96). But $\angle A = \angle B$ (Hyp.). $\therefore \angle DCE = \angle ECA$ (Ax. 1).

11. $\angle CBD = \angle BPR$ (§ 96). $\angle PBR = \angle CBD$ (Hyp.). $\therefore \angle BPR = \angle PBR$ (Ax. 1). $\therefore BR = PR$ (§ 115).

If from any point in the bisector of an angle a line is drawn parallel to one side of the angle, and meeting the other side, the triangle thus formed is isosceles.

12. $\angle ABC = \angle ACB$ (§ 82). $\therefore$ $\angle DBC = \angle ECB$ (§ 66). $\therefore$ $\triangle DBC = \triangle BCE$ (§ 79), etc.

13. Use § 66 (complements of equal ∡ are equal).

14. Denote the small ∡ to the left of the vertex by 1, 2; those to the right by 3, 4. Then $\angle 1 = \angle 4$ (§ 69); $\angle 2 = \angle 3$ (§ 69). But $\angle 3 = \angle 4$ (Hyp.). $\therefore$ $\angle 1 = \angle 2$ (Ax. 1).

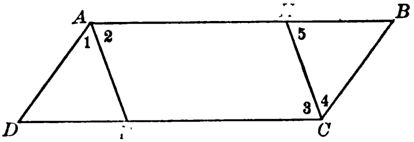
PAGE 96

1. Use figure of Ex. 2, p. 92 (text-book.) Show $\triangle DCE = \triangle BCA$ (§79). $\therefore$ $\angle E = \angle B$ (corr. ∡ of = ▲). $\therefore$ $DE \parallel BA$ (§ 89).

2. § 96, Ax. 5, § 89.

3. $\angle DCA = \angle A + \angle B$ (§ 103). $\therefore$ $2\angle DCE = 2\angle B$ (Ax. 9). $\therefore$ $\angle DCE = \angle B$ (Ax. 5). $\therefore$ $CE \parallel AB$ (§ 91).

4.



Given the ▱ $ABCD$, AF bisects $\angle DAB$, CH bisects $\angle DCB$.
To prove $AF \parallel HC$.
Proof. $\angle DAB = \angle DCB$ (§ 155); $\angle 2 = \angle 3$ (Ax. 5). But $\angle 3 = \angle 5$ (§ 96); $\angle 2 = \angle 5$ (Ax. 1). $\therefore$ $AF \parallel CH$ (§ 91).

5. Use the figure of Ex. 8, p. 93.
Given the lines AB and DC, AD and $BC \perp DC$, $AD = BC$.
To prove $AB \parallel DC$.
Proof. $AD \parallel BC$ (§ 92), $AD = BC$ (Hyp.). $DABC$ is a ▱ (§ 161).

PAGE 97

1. 57°; 85°.

2. 135°.

3. 125°; $180° - \frac{p° + q°}{2}$.

4. 70°; 20°; 50°.

5. 140°.

6. § 168. $\frac{(12-2)\,180°}{12}$, or 150°.

7. 5; 9; 35; $\frac{n(n-3)}{2}$. In general, in a polygon of n sides the number of diagonals $= {}_nC_2 - n = \frac{n(n-1)}{2} - n = \frac{n^2 - n - 2n}{2} = \frac{n(n-3)}{2}$. Or we may reason thus: from each vertex $n - 3$

diagonals may be drawn. ∴ from n vertices $n(n-3)$ diagonals are drawn; but each of these is used twice. The number of distinct diagonals is $\frac{n(n-3)}{2}$.

8. Denote the supplementary adj. $\measuredangle$ by $2\angle a$ and $2\angle b$. Then $2\angle a + 2\angle b = 180°$ (§ 68). ∴ $\angle a + \angle b = 90°$ (Ax. 5).

PAGE 98

9. Denote the $\parallel$ lines by AB and CD, the transversal by PQ, the bisectors by PR and QR. Then $\angle BPQ + PQD = 2$ rt. $\measuredangle$. (§ 98). ∴ $\angle RPQ + \angle PQR = 1$ rt. $\angle$ (Ax. 5). . . . (1)
But $\angle R + \angle RPQ + \angle PQR = 2$ rt. $\measuredangle$ (§ 102) . . . (2)
Subtract (1) from (2), $\angle R = 1$ rt. $\angle$ (Ax. 3).

10. $\angle BCA = \angle A$ (§ 82), $\angle BCD = \angle D$ (§ 82). ∴ $2\angle A + 2\angle D = 180°$ (§ 102). ∴ $\angle A + \angle D = 90°$, or $\angle BCA + \angle BCD = 90°$ (Ax. 9).

1. Let x be the complement. ∴ $3x =$ the $\angle$. ∴ $3x + x = 90°$. ∴ $x = 22\frac{1}{2}°$, $3x = 67\frac{1}{2}°$. *Ans.*

2. 90°; 45°. 3. 36°; 72°; 108°; 144°. 4. 30°; 60°; 90°.

5. Let $x =$ the less $\angle$. Then $x + 30° =$ the greater $\angle$. ∴ $x + x + 30° = 180°$. ∴ $x = 75°$; $x + 30° = 105°$. *Ans.*

6. $x + 2x + x + 2x = 360°$, etc.; 60°; 120°. *Ans.*

7. The vertex $\angle = 180° - 105° = 75°$. Hence, $x + 2x + 75° = 180°$, etc. 35°; 70°; 75°. *Ans.*

PAGE 99

8. 9. 9. 7; 12; 6.

10. Denote the number of sides by n. Then $\frac{2n-4}{n} = \frac{7}{4}$ (§ 168). ∴ $n = 16$. *Ans.*

11. The exterior $\measuredangle$ at the base are $180° - a$ and $180° - b$. The vertex $\angle$ of the △ is $180° - (a + b)$ (§ 102). $180° - a + 180° - b - (180° - a - b) = 180° - a + 180° - b - 180° + a + b = 180°$.

12. The exterior $\angle$ of the small △ on the base (to the right) is $b + x$. ∴ $a + x = b + x$ (§ 103). ∴ $a = b$ (Ax. 3).

13. In the original △ the $\measuredangle$ are b, x, x. ∴ $b + x + x = 180°$ (§ 102). ∴ $\angle b = 180° - 2x$. ∴ $\frac{1}{2}b = 90° - x$ (Ax. 5). But $a = 90° - x$ (§ 106). ∴ $a = \frac{1}{2}b$ (Ax. 1).

14. $x + y = 180°$ (Ax. 5). $\therefore$ each pair of opposite sides in the figure is $\parallel$ (§ 94), etc.

PAGE 100

1. Use § 83.

2. Use the figure of Ex. 14, p. 90 (omitting line BE). Draw $CF \parallel AB$. Then $BA = FC$ (§ 157). But $BA = CD$ (Hyp.). $\therefore CF = CD$ (Ax. 1). $\therefore \angle CFD = \angle D$ (§ 82). But $\angle A = \angle CFD$ (§ 97). $\therefore \angle A = \angle D$ (Ax. 1).

3. If the angles which the legs of a trapezoid make with a base of the trapezoid are equal, the trapezoid is isosceles.

Draw the same auxiliary line as in the preceding Ex. and reverse the process of proof.

Exs. 2 and 3 may also be proved by drawing $\perp$s from the extremities of the upper base to the lower base and using §§ 92, 157, 117, 111.

4. Draw a line through the vertex of $\angle b$ parallel with AB and use §§ 101, 96, and Ax. 2.

5. Draw the same auxiliary line as in Ex. 4.

6. Then prove $\triangle ADF = \triangle FDC$ by §§ 100, 63, 79.

7. Use the figure of Ex. 6 (text-book).

Let $\angle A = 2 \angle B$. Draw the median DC as an auxiliary line. Then $DA = DC$ (Ex. 6). $\therefore \angle DCA = \angle A$ (§ 82). Also $DB = DC$ (Ex. 6). $\therefore \angle DCB = \angle B$. Adding, $\angle A + \angle B = \angle DCA + \angle DCB = 90°$ (Ax. 2). $\therefore 3 \angle B = 90°$, or $2 \angle B = 60°$. $\therefore \angle DCA = 60°$ (Ax. 1). $\therefore \angle ADC = 60°$ (§ 102). $\therefore AC = AD$ (§ 115) $= \frac{1}{2}AB$ (Hyp.).

8. From P draw $PT \perp AD$. $\therefore PT \parallel BC$ (§ 92), $\angle APT = \angle B$ (§ 97). $\therefore \triangle ARP = \triangle ATP$ (§ 110). $\therefore PR = AT$. But $PQ = TD$ (§ 157). Use Ax. 2.

9. Denote the vertex between A and C by F, and the other vertex by H. Draw AC and BD. Then $ABDC$ is a ▱ (§ 161). $\therefore AC = BD$ (§ 155). $\therefore \triangle AFC = \triangle BHD$ (§ 83). $\therefore \angle F = \angle H$. Also $\angle ABD = \angle ACD$ (§ 155). $\angle HBD = \angle ACF$ (corr. $\angle$s of $=$ $\triangle$s). Adding, $\angle HBA = \angle FCD$ (Ax. 2), etc.

PAGE 101

10. Draw $DX \perp AC$, meeting AC in the point X, and draw $EY \perp AC$, and meeting AC produced at Y. Prove $\triangle ADX = \triangle CEY$ (§ 110). $\therefore \triangle DXF = \triangle FYE$ (§ 111), etc.

7. For if the two lines are parallel, the alt. int. $\angle$s are = (§ 96), which is contrary to the hypothesis. Hence the two given lines are not ||.

PAGE 102

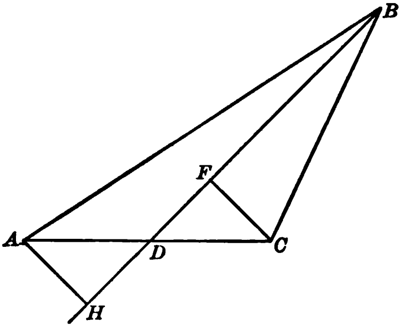
4. CPQ is a straight line (Constr.). $\therefore \angle BPQ = \angle CPA$ (§ 69). But $\angle BPD = \angle CPA$ (Hyp.) $\therefore \angle BPQ = \angle BPD$ (Ax. 1). $\therefore PQ$ coincides with PD. $\therefore PD$ is in the same straight line with CP (for it coincides with PQ, which is in the same straight line with CP).

5. Produce the bisector of one of the vertical $\angle$s and show that the bisector of the other vertical $\angle$ coincides with the produced line (use the same method of proof as in Ex. 4).

6. Let BE and DC intersect at the point F. If DC and BE bisect each other show $\triangle BFD = \triangle FEC$ (§ 79). $\therefore \angle FBD = \angle FEC$ (corr. $\angle$s of = $\triangle$s). $\therefore DB \parallel EC$ (§ 89), which is impossible, since AB and AC meet in the point A. $\therefore BE$ and DC cannot bisect each other.

1. (Group 24). Use the figure of Ex. 13, p. 90. Let $\angle BAC = \angle DAC$, and $\angle BCA = \angle DCA$; prove $\triangle ABC = \triangle ADC$. Use § 80.

2.



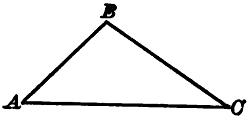
Given the $\triangle ABC$, the median BH, AH and $FC \perp BH$.
To prove $AH = FC$.
Proof. In the $\triangle$s AHD and DFC, $AD = DC$ (Hyp.). $\angle ADH = \angle FDC$ (§ 69). $\angle AHD = \angle CFD$ (§ 63). $\therefore \triangle AHD = \triangle DFC$ (§ 110). $\therefore AH = FC$ (corr. sides of = $\triangle$s).

3. Use the figure of Ex. 8, p. 90. Let ABC be the given $\triangle$, $CD \perp AB$, $BF \perp AC$; prove $\triangle DBC = \triangle BFC$ (§ 117), etc.

4. Denote the point of intersection by F. Then $AF + FC > AC$ (§ 78), $FD + FB > DB$ (§ 78). Add, etc.

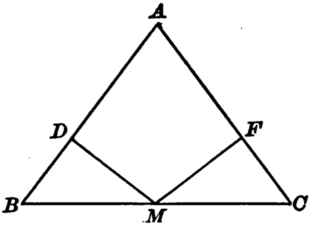
5. 68°; 34°; 102°; etc.

6.



Given AB any side of the $\triangle ABC$, and $AC > BC$.
To prove $AB > AC - BC$.
Proof. $AB + BC > AC$ (§ 78). Subtracting BC from each member of the inequality, $AB > AC - BC$ (Ineq. Ax. 1, § 133).

7.



Given $\triangle ABC$, $AB = AC$, $BM = MC$, $MD \perp AB$, $MF \perp AC$.
To prove $DM = MF$.
Proof. In the $\triangle$s BDM and MFC, $\angle B = \angle C$ (§ 82). $BM = MC$ (Hyp.). $\therefore \triangle BDM = \triangle MFC$ (§ 110). $\therefore DM = MF$, etc.

PAGE 103

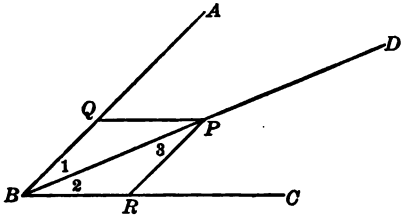
8. If the $\perp$s drawn from the midpoint of one side of a $\triangle$ to the other two sides are equal, then the other two sides are equal, and the $\triangle$ is isosceles. Use the figure of Ex. 7. Prove $\triangle BDM = \triangle MFC$ (§ 117). $\therefore \angle B = \angle C$. $\therefore AB = AC$ (§ 115).

9. Denote the vertex $\angle$ by $2x$. Then each base $\angle = 90° - x$ (§ 102). Hence an ext. $\angle$ at the base $= 2x + 90° - x$ (§ 103), etc.

10. Draw an auxiliary line from the vertex of $\angle a$ to the vertex of $\angle d$, and produce this line through vertex of $\angle d$. Use § 103 (twice) and add (Ax. 2).

11. $AC + CD > AD$ (§ 78), or $BC + CD > AD$ (Ax. 9). Also $\angle ACD$ is 120° (§ 103). $\therefore \angle ACD > \angle D$ (§ 105). $\therefore AD > AC$ (§ 135), or $AD > AB$ (Ax. 9).

12.



Given BD the bisector of the $\angle ABC$, $PQ \parallel BC$, $PR \parallel AB$.
To prove $PQBR$ a rhombus.
Proof. $\angle 1 = \angle 2$ (Hyp.), $\angle 1 = \angle 3$ (§ 96). $\therefore \angle 2 = \angle 3$ (Ax. 1). $\therefore BR = PR$ (§ 115). But $BQ = PR$ and $QP = BR$ (§ 155). $\therefore PR = BR = BQ = QP$ (Ax. 1).

13. Use the figure of p. 37 (text-book).
Given $\triangle ABC$, $BD \perp AC$, $AD = DC$.
To prove $AB = BC$.
Proof. Show $\triangle ADB = \triangle BDC$ by § 79, etc. Or use § 120.

14. $\angle B = \angle C$ (§ 82). $\therefore 2 \angle C + 4 \angle C = 180°$ (§ 102). $\therefore 6 \angle C = 180°$. $\therefore \angle C = 30°$ (Ax. 5). $\therefore \angle BED = 60°$ (§ 102). $\therefore \angle EAF = \angle B + \angle C = 2 \angle C = 60°$ (§ 103). $\therefore \angle EFA = 60°$ (§ 102).

15. Use the figure of Ex. 6, p. 94.
Given the quadrilateral $ABCD$, in which the diagonals AC and BD intersect at rt. $\angle$s in the point O, $AO = OC$, and $BO = OD$.
To prove $ABCD$ a rhombus.
Proof. In the $\triangle$s AOB and BOC, $AO = OC$ (Hyp.), $BO = BO$ (Ident.), $\angle BOA = \angle BOC$ (§ 63). $\therefore \triangle AOB = \triangle BOC$ (§ 79). $\therefore AB = BC$ (corr. $\angle$s of $=$ $\triangle$s). In like manner it can be proved that $BC = CD = AD$. Hence $ABCD$ is a $\square$ (§ 160), and a rhombus (§ 148). If the diagonals of the given figure are equal the figure may be shown to be a square.

16. Use the figure of Ex. 8, p. 90. Let DC and BF intersect at the point O and draw OA. Prove $\triangle DBC = \triangle BFC$ (§ 110). $\therefore \angle OBC = \angle OCB$. $\therefore BO = OC$ (§ 115). Hence prove $\triangle BOA = \triangle AOC$ (§ 83), etc.

17. See figure, p. 64 (text-book). Let $PQ \perp AB$, $PR \perp BC$. Then, in the quadrilateral $PQBR$, $\angle PQB + \angle QBR + \angle BRP + \angle$

$RPQ' = 4$ rt. $\measuredangle$ (§ 167). But $\angle PQR + \angle PRB = 2$ rt. $\measuredangle$ (Hyp.) Subtract, etc.

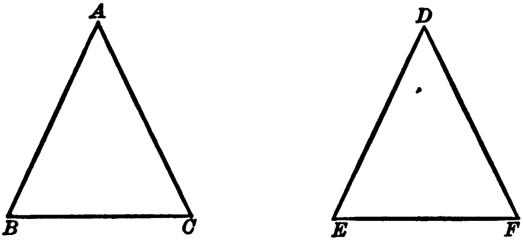
18. $AP \parallel QC$ (Hyp. and § 146), $AB = DC$ (§ 155). $\therefore AP = QC$ (Ax. 3). $\therefore APCQ$ is a $\square$ (§ 161).

19. Use figure of Ex. 8, p. 93. **Prove** $\triangle ADB = \triangle ACD$ (§ 83). $\therefore \angle BAD = \angle CDA$ (corr. $\measuredangle$ of $=$ $\blacktriangle$). But $\angle BAD + \angle CDA = 2$ rt. $\measuredangle$ (§ 98). $\therefore 2 \angle BAD = 2$ rt. $\measuredangle$ (Ax. 9). $\therefore \angle BAD = 1$ rt. $\angle$. In like manner it may be shown that the other $\measuredangle$ of $ABCD$ are rt. $\measuredangle$. Hence $ABCD$ is a rectangle (§ 149).

PAGE 104

20. AC. For $\angle BCA = 60°$ (§ 64). $\therefore \angle B = 70°$ (§ 102). Use § 135.

21.



Two cases arise according as the given $\angle$ is a base $\angle$ or a vertex $\angle$.

I. **Given** $\blacktriangle$ ABC and DEF, $BC = EF$, $AB = AC$, $DE = DF$, $\angle B = \angle E$.

To prove $\triangle ABC = \triangle DEF$.

Proof. $\angle C = \angle B$ (§ 82), $\angle F = \angle E$ (§ 82). But $\angle B = \angle E$ (Hyp.). $\therefore \angle C = \angle F$ (Ax. 1). Hence, prove $\triangle ABC = \triangle DEF$ (§ 80).

II. **Given** ABC and DEF isosceles $\blacktriangle$ in which the base $BC =$ the base EF and $\angle A = \angle D$.

To prove $\triangle ABC = \triangle DEF$.

Proof. Show that $\angle B = \frac{1}{2}(180° - \angle A)$ (§ 102). Also, $\angle E = (180° - D)$ (§ 102). $\therefore \angle B = \angle E$ (Ax. 1), etc. Use § 80.

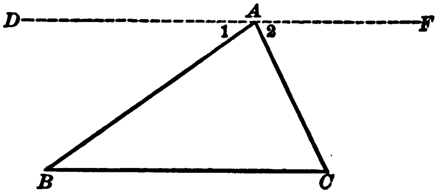
22. Use §§ 111, 102, 115, etc.

23. Produce AP to meet BC at Q.

Then $AB + BQ > AP + PQ$ (§ 78); $PQ + QC > PC$ (§ 78). $\therefore AB + BQ + PQ + QC > AP + PQ + PC$ (Ineq. Ax. 1). $\therefore AB + BQ + QC > AP + PC$ (Ineq. Ax. 1), etc.

24. $\angle APC > \angle PQC > \angle B$ (§ 87). $\therefore \angle APC > \angle B$ (§ 133, Ineq. Ax. 4).

25.



Given the $\triangle ABC$.
To prove $\angle B + \angle BAC + \angle C = 2$ rt. $\angle$s.
Proof. Draw DF through $A \parallel BC$. Then $\angle 1 + \angle BAC + \angle 2 = 2$ rt. $\angle$s (§ 68). But $\angle 1 = \angle B$, $\angle 2 = \angle C$ (§ 96). Substituting, $\angle B + \angle BAC + \angle C = 2$ rt. $\angle$s (Ax. 9).

26. $AC = BC$ (Hyp.), $RC = BQ$ (Hyp.), $AR = QC$ (Ax. 3). $\angle A = \angle C$ (§ 82). $\therefore \triangle APR = \triangle QRC$ (§ 79). $\therefore PR = QR$. In like manner show $PR = PQ$, etc.

27. Denote the point of intersection of the bisectors by R. Then show the $\triangle APR$ to be isosceles (§ 96, Ax. 1, § 115). $\therefore AP = PR$. In like manner show that $\triangle RQC$ is isosceles and $RQ = QC$. Use Ax. 2.

28. Use superposition. Or, draw a pair of corresponding diagonals and use § 79, Ax. 3, and § 79 again.

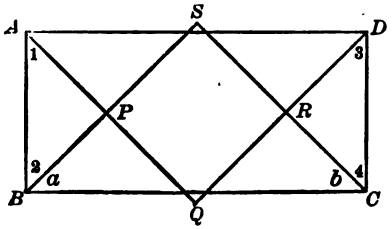
29. See Ex. 17, p. 86.

PAGE 105

30. See Ex. 17, p. 86.

31. Use Ex. 9, p. 98, and § 92.

32. Use Ex. 31. Then show that the sides of the rectangle are equal.

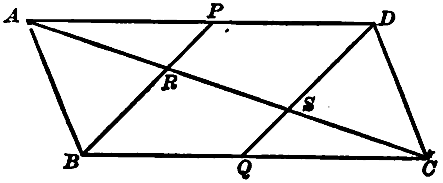


Thus, on the figure, $\angle 1 = \angle 2 = \angle 3 = \angle 4$ (each $= \frac{1}{2}$ a rt. $\angle$). $AB = CD$ (§ 155). $\therefore \triangle ABP = \triangle CRD$ (§ 119).

$\therefore BP = RC$ (corr. sides of = ▲). But $\angle a = \angle b$ (each = ½ a rt. $\angle$). $\therefore BS = CS$ (§ 115). $\therefore PS = SR$ (Ax. 3).

33. §§ 101, 155, 83.

34.



Given $ABCD$ a ▱, $AP = PD$, $BQ = QC$.
To prove $AR = RS = SC$.
Proof. $PD = BQ$ (Ax. 5). $\therefore PDQB$ is a ▱ (§ 161). In $\triangle ADS$, $AR = RS$ (§ 169). Also in $\triangle CRB$, $RS = SC$ (§ 169). $\therefore AR = RS = SC$ (Ax. 1).

35. $\triangle ABP = \triangle CQD$ (§ 79). $\therefore \angle BPQ = \angle DQP$ (§ 66). $\therefore BP \parallel QD$ (§ 89). Use § 161. Six.

36. Use Ex. 2, p. 100, and § 98.

37. Use Ex. 2, p. 100, and § 79.

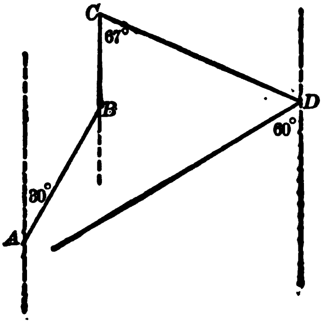
1. $22\frac{1}{2}°$; $67\frac{1}{2}°$; $146\frac{1}{4}°$; $11\frac{1}{4}°$.

2. By the law of reflection, $\angle POA' = \angle ROB'$; or, $90° - x = \angle POB - y + \angle BOB'$. $\therefore 90° - x = 90° - y + x$. $\therefore y = 2x$.

PAGE 106

3. See Ex. 8, p. 83.

5.



7. But $a + b + c = 180°$ (§ 102). $\therefore c = 90°$ (Ax. 3).

PAGE 107

1. §§ 57, 59.
2. §§ 15, 96, 97, 98, 100, 101, 157, 158, 169.
3. §§ 55, 56, 58, 69, 99.
4. §§ 60, 61, 63, 64, 65, 66, 67, 68, 69, 112, 113, 114, 126.
5. §§ 70, 78, 87, 102, 103, 104, 105, 134, 135.
6. §§ 79, 80, 83, 107, 109, 110, 111, 117, 138, 139.
7. §§ 142, 160, 161, 162.
8. §§ 146, 155, 156, 159, 167.
9. § 148, also see Ex. 6, p. 82.

PAGE 108

10. See §§ 73, 74.
11. See §§ 73, 74.
12. Make the three angles of one △ = the corresponding angles of the other, but the corresponding sides unequal.
13. When two straight lines intersect after one of the angles of intersection has been measured, Prop. I enables us to determine the other angles without the labor of measuring them, etc.

15. § 172.
16. § 174.

PAGE 113

1. Prop. II enables us to determine without effort the equality of two arcs in a circle or equal circles, when the central angles which these arcs subtend are equal, etc.

PAGE 114

1. 1¼ in.

PAGE 115

1. 1 in.
2. Prop. V enables us to determine without effort the equality of two chords in a circle or equal circles, when the arcs subtended by these chords are known to be equal.
3. No.

PAGE 117

1. $\therefore \overset{\frown}{AB} = \overset{\frown}{CD}$ (§ 198). To each of these add $\overset{\frown}{BC}$, etc.
2. $\therefore \overset{\frown}{PR} = \overset{\frown}{QS}$ (§ 198). $\therefore \overset{\frown}{PQ} = \overset{\frown}{RS}$ (Ax. 3). $\therefore$ Chord PQ = chord RS (§ 200), etc.

3. $\overset{\frown}{AC} > \overset{\frown}{BD}$ (Hyp.). Subtract $\overset{\frown}{BC}$ from each. $\therefore \overset{\frown}{AB} > \overset{\frown}{CD}$ (Ineq. Ax. 1). $\therefore$ chord $AB >$ chord CD (§ 201).

4. If A, B, C, and D are four points taken in succession on a semicircle, and chord $AB >$ chord CD, then $\overset{\frown}{AC} > \overset{\frown}{BD}$.

For $\overset{\frown}{AB} > \overset{\frown}{CD}$ (§ 201). Add $\overset{\frown}{BC}$ to each, etc.

8. 120°.

PAGE 120

3. Construct an arc of 45° and bisect it.

4. The point F; no.

PAGE 121

1. $\frac{1}{3}$ sec.; $\frac{2}{3}$ sec.

PAGE 125

1. Then $SO = OR$ (§ 207). Prove $\triangle SPO = \triangle OPR$ by § 117.

2. Use § 207. **3.** No. Use § 78.

4. Use the figure of p. 121 (text-book). Chord $AB =$ chord CD (§ 207).

$\therefore \overset{\frown}{AB} = \overset{\frown}{CD}$ (§ 198).

5. $\angle C + \angle D$ is suppl. $\angle COD$ (§ 102);
$\angle g + \angle h$ is suppl. $\angle COD$ (§ 68);
$\therefore \angle C + \angle D = \angle g + \angle h$ (§ 66). $\therefore 2 \angle D = 2 \angle h$ (Ax. 9), etc.

6. The tangents are $\perp$ the diameter (§ 210), and $\therefore \parallel$ (§ 92).

7. Use § 121.

PAGE 127

1. 4. **2.** An infinite number. **3.** The circles intersect.

PAGE 129

1. Use § 78. **2.** Ⓢ touching externally.

3. One ⊙ outside the other.

4. Intersecting Ⓢ or one ⊙ within the other.

5. Intersecting Ⓢ. **6.** No. **7.** Yes.

8. One circle is wholly within the other.

PAGE 130

1. One internal, two external. **3.** Two external.

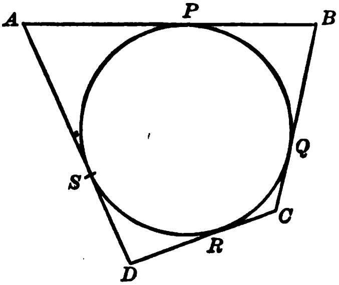
2. One external. **4.** None.

5. Two internal and two external.

1. (Group 30). Use the figure, p. 127 (text-book). Draw AB. $PA = PB$ (§ 216), $OA = OB$ (§ 188), $PO \perp AB$ (§ 121).
2. Use figure 3, p. 142 (text-book). $PA = PB$ (§ 216). $\therefore \angle PAB = \angle PBA$ (§ 82). $\therefore \angle PAB = \frac{1}{2}(180° - 60°) = 60°$ (§ 102), etc.

PAGE 131

3. $RP = RA$ (§ 216), $PQ = QB$ (§ 216). $\therefore RP + PQ = RA + QB$ (Ax. 2).
4.



On the figure,

$$\left.\begin{aligned} PA &= SA, \\ PB &= QB, \\ RD &= SD, \\ RC &= QC. \end{aligned}\right\} \text{(§ 216).}$$

Adding, $AB + CD = AD + BC$ (Ax. 2).

5. Use the method of Ex. 4.
6. Use the method of Ex. 4.
7. In the figure of Ex. 4, if $ABCD$ becomes a ▱, $AB = DC$, and $BC = AD$ (§ 155). $\therefore AB + AB = AD + AD$ (Ex. 4 and Ax. 9). $\therefore 2\,AB = 2\,AD$. $\therefore AB = AD$ (Ax. 5), etc.
8. $AP = PC$ (§ 216), $PB = PC$ (§ 216). $\therefore AP = PB$ (Ax. 1).
9. Use same method of proof as in Ex. 8.
10. $OA = OP$ (§ 188). $\therefore \angle OAP = \angle OPA$ (§ 82). Also $O'B = O'P$ (§ 188). $\therefore \angle O'BP = \angle O'PB$ (§ 82). $\therefore \angle OAP = \angle O'BP$ (Ax. 1). $\therefore OA \parallel O'B$ (§ 91).

PAGE 132

12. PC is 160 yd. (§ 216). Erect $\perp$s to AB and CD at A and C. Use § 212.

PAGE 133

4. Yes.
5. No.

PAGE 135

1. $\frac{1}{2}$. 2. $\frac{20}{25}$ or $\frac{4}{5}$.

3. Take a given line as a radius, and the ends of the line as a center in turn.

4. Use each vertex of the △ as a center and a side as the radius.

5. Use each vertex of an equilateral △ as a center and one half the side of the △ as a radius.

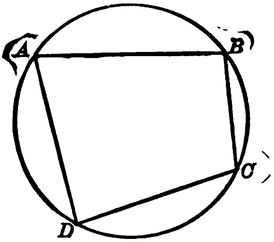
PAGE 138

1. 46°; 134°. 2. 60°; 30°.

3. $\angle DAC = 25\frac{5}{7}°$; $\angle DAB = 12\frac{6}{7}°$.

PAGE 139

4.



On the figure, $\angle B \overset{m}{=} \frac{1}{2}\overset{\frown}{ADC}$ (§ 235). Also $\angle D \overset{m}{=} \frac{1}{2}\overset{\frown}{ABC}$ (§ 235). Adding, $\angle B + \angle D \overset{m}{=} \frac{1}{2}$ the circle $ABCD$ (Ax. 2). $\therefore \angle B + \angle D = 180°$, etc.

5. 360°. 6. $\angle S = 52\frac{1}{2}°$, etc. 7. 540°.

8. $\angle APB = 90°$ (§ 238). Prove $\triangle APB = \triangle PBC$ (§ 79).

9. $37\frac{1}{2}°$. 10. Use § 238.

11. Then at one end of the diameter construct an angle equal to the given acute angle, etc. Or at one end of the hypotenuse construct an ∠ = the given acute ∠ (§ 86). From the other end of the hypotenuse draw a ⊥ to the other side of the acute ∠ (§ 129), etc.

12. On the given hypotenuse as a diameter construct a semicircle (§ 128, Post. 3). With one end of the given hypotenuse as a center and the given leg as a radius, describe an arc intersecting the semicircle. From the point of intersection draw lines to the extremities of the given hypotenuse. Use § 238.

13. Construct a circle and in it draw a chord smaller than the radius. At one end of the chord construct an angle of 120° (twice 60°), etc.

PAGE 140

1. 66°; 114°.
2. $124° = \frac{1}{2}(52° + \overset{\frown}{CB})$. $\therefore$ 196°.. *Ans.*
3. 34°.

PAGE 141

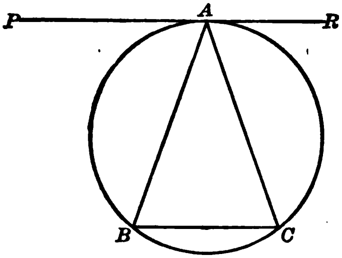
1. 47°.
2. $\angle CPD = 40°$; $\angle CPA = 50°$, etc.
3. $\overset{\frown}{PDC} - (360° - \overset{\frown}{PDC}) = 128°$. $\therefore$ $\overset{\frown}{PDC} = 244°$, etc.

PAGE 143

1. $\angle P = 37°$, etc.
2. Denote the $\overset{\frown}{AFC}$ by x. Then $80° = \frac{1}{2}(360° - x - x)$. $\therefore$ $x = 100°$.

PAGE 144

1. §§ 200, 202, 207, 216.
2. (1) §§ 196, 237, 241, 242; (2) See Ex. 4, p. 139; (3) §§ 210, 238.
3. §§ 195, 198, 202, 235, 244.
4. Use figure 3, p. 142 (text-book). $\angle PAB \overset{m}{=} \frac{1}{2}\overset{\frown}{AFB}$ (§ 241). Also $\angle PBA \overset{m}{=} \frac{1}{2}\overset{\frown}{AFB}$. $\therefore$ $\angle PAB = \angle PBA$ (Ax. 1).
5.



Let ABC be an inscribed △ in which $AB = AC$, and let PR be tangent to the ⊙ at A. Then $\overset{\frown}{AB} = \overset{\frown}{AC}$ (§ 198). $\angle PAB = \angle RAC$ (measured by = arcs, § 241). But $\angle ABC \overset{m}{=}$

$\frac{1}{2}\overset{\frown}{AC}$ (§ 235). $\therefore \angle ABC = \angle RAC$ (Ax. 1). $\therefore PR \parallel BC$ (§ 89). The converse is true.

6. On the figure, p. 140 (text-book), if $\angle DPB$ should be made a rt. $\angle$, $\angle DPB \overset{m}{=} \frac{1}{2}(\overset{\frown}{DB} + \overset{\frown}{AC})$ (§ 240). $\therefore \frac{1}{2}(\overset{\frown}{DB} + \overset{\frown}{AC}) = 90°$. $\therefore \overset{\frown}{DB} + \overset{\frown}{AC} = 180°$ (Ax. 4).

7. $\angle BAC \overset{m}{=} \frac{1}{2}\overset{\frown}{AB}$ (§ 241). But $\angle AOB \overset{m}{=} \overset{\frown}{AB}$. $\therefore \frac{1}{2}\angle AOB \overset{m}{=} \frac{1}{2}\overset{\frown}{AB}$ (Ax. 5). $\therefore \angle BAC = \frac{1}{2}\angle AOB$ (Ax. 1).

8. On the figure to Ex. 4, p. 139, produce DC through C to F. Then $\angle BCF$ is supplement of $\angle BCD$ (§ 32). But $\angle DAB$ is supplement of $\angle BCD$ (Ex. 4, p. 139). $\therefore \angle A = \angle BCF$ (§ 66).

9. Use § 238.

10. Use the figure of Ex. 18, p. 145, as given in the Key (omitting the circumscribed rhombus). Then chord $AD \parallel$ chord BC. $\overset{\frown}{AB} = \overset{\frown}{DC}$ (§ 244). In like manner $\overset{\frown}{BC} = \overset{\frown}{AD}$. Adding, $\overset{\frown}{ABC} = \overset{\frown}{ADC}$. $\therefore \overset{\frown}{ABC}$ = semicircle. $\therefore \angle B =$ rt. $\angle$ (§ 238). In like manner $\angle$s A, C, D are rt. $\angle$s, etc.

11. Use § 235.

12. Then $\overset{\frown}{ABC} = \overset{\frown}{BCD} = \overset{\frown}{CDE}$, etc. (Ax. 4). $\therefore \angle ABC = \angle BCD = \angle CDE$, etc. (§ 237).

PAGE 145

13. §§ 97, 230, 235.

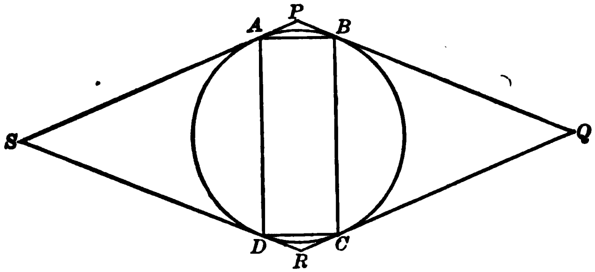
14. $\angle B = \angle D$ (each $\overset{m}{=} \frac{1}{2}\overset{\frown}{AC}$, § 235). Use § 107.

15. $\overset{\frown}{AD} = \overset{\frown}{BE}$ (§ 244). $\therefore \angle CAD = \angle AED$ (§§ 241, 235, Ax. 1). Also, $\angle C \overset{m}{=} \frac{1}{2}(\overset{\frown}{ABE} - \overset{\frown}{AD})$, or $\frac{1}{2}\overset{\frown}{AB}$ (§ 242). $\therefore \angle C = \angle AEB$ (§ 235, Ax. 1). Use § 107.

16. Draw $\perp$s from the centers of the circles to the cutting line. These $\perp$s are $\parallel$ (§ 92), and $\therefore =$ (§ 157). Then use § 207.

17. $\angle BAE = \angle BCD$ (§ 114). $\therefore \overset{\frown}{BD} = \overset{\frown}{BE}$, since they measure equal $\angle$s.

18.

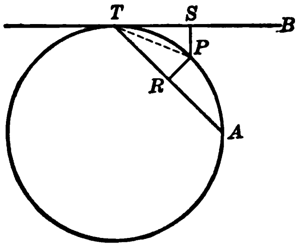


In the figure $AB \parallel CD$ (Hyp.). $\therefore \overset{\frown}{BC} = \overset{\frown}{AD}$ (§ 244). $\therefore \angle SAD = \angle SDA = \angle QBC = \angle QCB$ (§ 241). Chord BC = chord AD (§ 155). $\therefore \triangle SAD = \triangle QBC$ (§ 80). $\therefore BQ = SA$ (corr. sides of = ▲).

But $AP = BP$ (§ 216). Adding, $SP = PQ$ (Ax. 2). In like manner, $PQ = QR = RS$, etc. Use § 160.

1. Draw OB. Then $\angle AOB = \angle BOC$ (§ 196). $\therefore \triangle AOB = \triangle BOC$ (§ 110), etc.

2.



On the figure, $\overset{\frown}{PT} = \overset{\frown}{AP}$ (Hyp.). Draw PT. Then $\angle ATP = \angle PTS$ ($\overset{m}{=}$ ½ of equal arcs AP and TP, §§ 235, 241). $\therefore \triangle RPT = \triangle STP$ (§ 110), etc.

3. Produce OD to meet the circle at F. Then $\overset{\frown}{AF} = \frac{1}{2} \overset{\frown}{AC}$ (§ 202). $\therefore \angle ABC \overset{m}{=} \overset{\frown}{AF}$ (§ 235). But $\angle AOF \overset{m}{=} \overset{\frown}{AF}$ (§ 230), etc.

4. §§ 110, 207. The converse is true.

PAGE 146

5. Draw a line from the center to the point of intersection. Use §§ 204, 117.

6. Use the figure, p. 127 (text-book) and draw AB. Then $\angle APO$ = complement of $\angle AOP$ (§§ 210, 102). Also $AB \perp OP$ (Ex. 1, Group 30, p. 130). $\therefore \angle OAB$ is complement $\angle AOP$ (§ 102). $\therefore \angle OAB = \angle APO$ (§ 66). But $\angle APB = 2 \angle APO$, etc.

7. Draw the auxiliary lines as indicated in the figure. Prove the two $\triangle$ formed, equal by § 198, Ax. 3, §§ 200, 237, 80.

8. Draw the chord AC. Then $\overset{\frown}{AB} = \overset{\frown}{CD}$ (§ 198). $\overset{\frown}{BD} = \overset{\frown}{BD}$ (Ident.). Adding, $\overset{\frown}{ABD} = \overset{\frown}{CDB}$ (Ax. 2). $\therefore \angle A = \angle C$ (measured by equal arcs, § 235). $\therefore PA = AC$ (§ 115).

9. On the diagram, p. 127 (text-book), prove $\angle POA = \angle POB$. Denote the equal $\angle$ at the center of the diagram for Ex. 9 by a, a; b, b; c, c; d, d. Then $2a + 2b + 2c + 2d = 360°$ (§ 67). $\therefore a + b + c + d = 180°$ (Ax. 5), etc.

10. $\angle APM = 60°$ (§ 235, for $\overset{\frown}{AC} = 120°$). $\therefore \triangle PAM$ is equilateral (§ 102). $\therefore PA = AM$. But $AB = AC$ (Hyp.). $\therefore \angle PAB = \angle MAC$ (each $60° - \angle BAM$). Use § 80, etc.

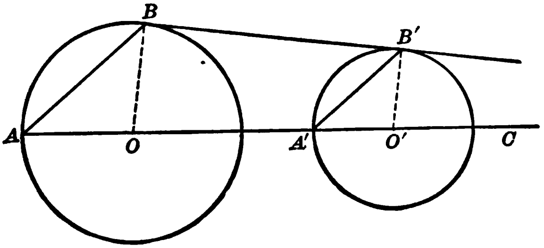
11. Show that the radii, given and drawn, form two pairs of equal $\angle$, and denote these $\angle$ by a, a and b, b. Then $\angle a + \angle b = 90°$ (Hyp.). $\therefore 2 \angle a + 2 \angle b = 180°$. Hence, two of these radii form a straight line. (§ 65). Use §§ 210, 92.

12. Draw a $\perp$ from the common center to AD. Use § 202 and Ax. 3.

13. $\angle PAB = \angle PBA$ (§ 82). $\therefore \angle x = \angle y$ (§ 66). $\therefore \overset{\frown}{BDC} = \overset{\frown}{ACD}$ (§ 235). $\therefore \overset{\frown}{AC} = \overset{\frown}{BD}$ (Ax. 3). Use §§ 200, 207, etc.

PAGE 147

14.



Draw radii OB and $O'B'$ from the centers O and O' to the point of tangency B and B'.

∴ OB and $O'B'$ are $\perp BB'$ (§ 210). ∴ $OB \parallel O'B'$ (§ 92). $\angle BOA' = \angle B'O'C$ (§ 97). But $\angle A = \angle ABO$ (§ 82). ∴ $\angle BOA' = 2 \angle A$ (§103). Similarly, $\angle B'O'C = 2 \angle B'A'O'$. ∴ $2 \angle A = 2 \angle B'A'O'$ (Ax. 1). ∴ $\angle A = \angle B'A'O'$ (Ax. 5). ∴ $AB \parallel A'B'$ (§ 97).

15. §§ 238, 202.

16. Denote the tangent by RS. $\angle D = \angle SPB = \angle APR = \angle C$ (§§ 235, 241, 69, Ax. 1). Use § 89.

17. Draw the common chord. Use Ex. 4, p. 139, §§ 64, 66, 94.

18. The circle will pass through both the center of the square and the vertex of the rt. △ (§ 238 used conversely). Use §§ 198, 235.

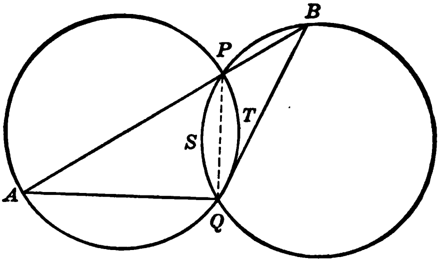
19. Through P draw a common tangent intersecting AB in R. Then (Ex. 8, p. 131), $RA = RP = RB$. ∴ describe a ⊙ with R as center and RA as radius. Use § 238.

20. Use § 238 conversely and § 237.

21. Draw the auxiliary line PQ. Then $\angle PQA = 1$ rt. $\angle = \angle PQB$ (§ 238). ∴ AQB is a straight line (§ 65).

PAGE 148

22.



Draw the common chord PQ. Then $\overset{\frown}{PSQ} = \overset{\frown}{PTQ}$ (§ 198). ∴ $\angle B = \angle A$ (measured by $\frac{1}{2}$ equal arcs, PSQ and PTQ, § 235). ∴ $QA = QB$ (§ 115).

23. The number of angular degrees in the inscribed $\angle ABC$ is one half the number of degrees of arc in $\overset{\frown}{AC}$.

1. The diameter through the given point is greatest chord (§ 194). The chord $\perp$ this diameter is least. For draw any other chord through the point and a $\perp$ to this chord from the center. Use §§ 136, 209.

2. $PO - CO < PC$ (Ex. 6, group 24, p. 102). $\therefore PA < PC$ (Ax. 9). Also $PO + OD > PD$ (§ 78). $PB > PD$ (Ax. 9).

3. $OC < OP + PC$ (§ 78), $OP + PB < OP + PC$ (Ax. 9). $\therefore PB < PC$ (Ineq. Ax. 1). Also $OD + OP > PD$ (§ 78). $\therefore AP > PD$ (Ax. 9).

4. From O' draw a line $\parallel CD$ to meet OR in T. Then $O'T \perp OR$ (§ 100). $\therefore OO' > O'T$ (§ 135). But $O'T = RS$ (§ 157). $\therefore OO' > RS$ (Ax. 9). $\therefore AB > CD$ (§ 202, Ax. 2, Ineq. Ax. 2).

5. This circle will pass through the point B (§ 238 used conversely). Let this $\odot$ cut OP at point R. $\therefore \angle OAB = \angle ORB$ (§ 237). But $\angle ORB > \angle P$ (§ 87), etc.

PAGE 149

1. For if it is not, on the chord opposite the rt. $\angle$, as a diameter, describe a semicircle. Either this semicircle will cut the sides of the given $\angle$, or one of the sides produced cuts the semicircle. In either case use § 87.

. If they are not diameters, the $\measuredangle$ of the rectangles are not right $\measuredangle$ (§ 239), etc.

3. If $\overset{\frown}{AB}$ is not $> \overset{\frown}{DF}$, it must $= \overset{\frown}{DF}$, or be less than $\overset{\frown}{DF}$, etc. (See method of proof in § 135.)

4. For the longer $\perp$ must be $\perp$ the other chord (§ 100) and bisect it (§ 202). $\therefore$ it coincides with the other $\perp$ (§ 93), etc.

5. Use § 87 and Ex. 4, p. 139.

1. (Group 36). $\angle P$ is measured by $\frac{1}{2}\overset{\frown}{BC}$ (constant). $\therefore \angle P$ is constant. But $\angle A$ is constant by hyp. $\therefore \angle P + \angle A$ is constant.

2. $RQ = AR + QB$. To each of these equals add $TR + TQ$. $\therefore RQ + TR + TQ = TA + TB$ (a constant sum).

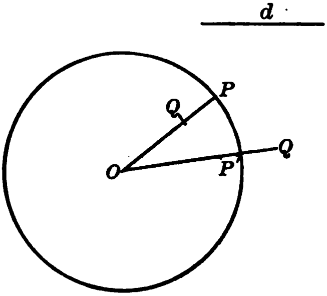
3. See Ex. 9, p. 146. $\angle AOR = \angle POR$. $\angle BOQ = \angle POQ$. Adding, $\angle AOR + \angle BOQ = \angle POR + \angle POQ = \angle ROQ$. $\therefore \angle ROQ = \frac{1}{2} \angle AOB$. But $\angle AOB$ is constant. $\therefore \angle ROQ$ is constant.

PAGE 150

4. $\angle CBD = \angle P + \angle PCB$ (§ 103). $\angle P$ is measured by $\frac{1}{2}\overset{\frown}{AB}$ (in its circle). $\angle PCB$ is measured by $\frac{1}{2}\overset{\frown}{AB}$ (in its circle), etc.

5. CD is constant (Hyp.). $OP = \frac{1}{2} CD$ (Ex. 6, p. 100). $\therefore OP$ is constant.

6. In order to give a convenient formal proof of this and the remaining exs. in this group it is best to give the following
Lemma. The locus of a point moving so as to be always at a given distance from a given fixed point is a circle whose center is the given point and whose radius is the given distance.



Given the point O, the line (segment) d, and the circle PP' whose center is O and radius $= d$.

To prove PP' the locus of the point moving so as always to be at distance d from O.

Proof. Let P be any point on the given circle. Then $PO = d$ (§ 188). Hence, every point on the given circle is at the given distance from O. Again, let Q be any point not on the given circle. Draw OQ. Then either OQ intersects the given circle at some point P', or on being produced OQ meets it at P. In the first case $OQ > OP'$, *i.e.*, $OQ > d$ (Ax. 8). In the second case $OQ < OP$; that is, $OQ < d$ (Ax. 8). Hence, every point not on the given circle is not at the distance d from O. Hence, the circle PP' is the required locus (§ 123).

To prove Ex. 6, the moving point is at the distance $r + a$ or $r - a$ from the center of the given circle (Hyp.). Hence, its locus is the two circles whose common center is the center of the given circle and whose radii are $r + a$ and $r - a$ respectively (Lemma). In case $a > r$ the second part of the answer disappears.

7. The moving point is always at a distance $\frac{1}{2}r$ from the center of the given circle (Hyp.); hence, its locus is the circle whose center is the center of the given circle and whose radius $= \frac{1}{2}$ radius of given circle (Lemma of Ex. 6).

8. The midpoints of all the given chords are equidistant from the center of the given circle (§§ 203, 207). Hence the locus of the moving point is a circle whose center, etc. (Lemma of Ex. 6).

9. Draw a line from the vertex of one of the right ▲ to the midpoint of the given hypotenuse. The line thus drawn = ½ the hypotenuse (Ex. 6, p. 100). Hence, the locus of the vertex of the given right ∠ is the circle having the given hypotenuse for is its diameter (Lemma of Ex. 6).

10. Ex. 10 reduces to Ex. 9 (§ 203). Hence the locus is the circle having for its diameter that radius of the original circle which is drawn to the given point on the circle.

11. The circle whose center is *O* and radius = ½*CD* (see Ex. 6, p. 100 and Lemma of Ex. 6).

12. Let *O* be the center of the given circle. On *OB* lay off $OC = QP$. Draw *OQ* amd *CP*. Then *C* is a fixed point, since *QP* is a constant. Also *OQPC* is a ▱ (§ 161); in every position of *P*, $CP = OQ$ = radius of given circle (§ 155). Hence the locus of *P* is a circle having *C* for its center and *OQ* for its radius (Lemma of Ex. 6).

PAGE 151

1. See figure of Ex. 2, p. 145. Let *TB* be a tangent, *TA* a chord, $\angle ATP = \angle PTB$, and prove $\overset{\frown}{TP} = \overset{\frown}{PA}$. Use §§ 235 and 241.

2. §§ 244, 200.

3. $\overset{\frown}{AC} = 35°$; $\angle T = 100°$; $\angle TAB = \angle TBA = 40°$; $\angle TBC = 57\frac{1}{2}°$. $\angle BAD = 47\frac{1}{2}° = \angle BCD$; $\angle S = 30°$; $\angle ABC = \angle ADC = 17\frac{1}{2}°$. If *AD* and *BC* intersect at *O*, $\angle BOD = \angle AOC = 65°$. $\angle BOA = \angle COD = 115°$.

4. By § 246, the ∥ tangents bisect the circle at their points of tangency. Hence, the line joining the points of contact is a diameter (§ 192).

5. Use Ex. 7, p. 131.

6. $OB = BP$ (Hyp.). $\therefore \angle ABO = 2 \angle P$ (§§ 103, 82, Ax. 9). But $\angle A = \angle OBA$ (§ 82). $\angle AOC = \angle A + \angle P$ (§ 103) $= 2 \angle P + \angle P = 3 \angle P$.

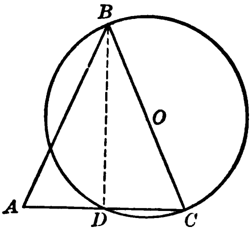
7. By § 198 each side of the square subtends an arc of 90°. ∴ required ∠ = 45° (§ 241).

8. Draw ⊥s from the center upon the given secants. The two rt. ▲ thus formed are = (§ 110). ∴ chords are = (§ 207). Add equal semichords to equal corresponding sides of ▲.

9. Use Ex. 5, p. 149, §§ 64, 167.

10. Draw the line of centers. Prove that this passes through the point of contact of the two circles. Then use §§ 69, 82, Ax. 1, § 89.

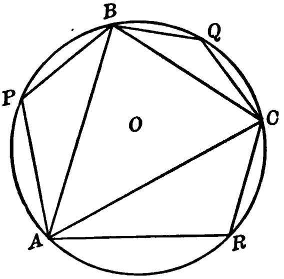
11.



Let ABC be a triangle in which $AB = BC$ and O a circle having BC for its diameter and intersecting AC in D. Then BDC is a right ∠ (§ 238). ∴ △ ABD = △ DBC (§ 117). ∴ $AD = DC$.

12. Draw a line from one end of the chord to the midpoint of the arc. Use §§ 235, 241, Ax. 1, § 89.

13.



Given the circle O with the inscribed △ ABC, P any point on $\overset{\frown}{AB}$, Q on $\overset{\frown}{BC}$, R on $\overset{\frown}{AC}$.

To prove ∠ APB + ∠ BQC + ∠ ARC = 4 rt. ∡.

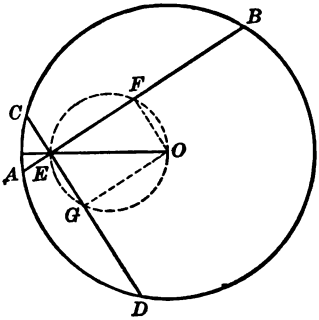
Proof. $\angle APB \overset{m}{=} \frac{1}{2}(\overset{\frown}{BQC} + \overset{\frown}{ARC})$ (§ 235). $\angle ARC \overset{m}{=} \frac{1}{2}(\overset{\frown}{BQC} + \overset{\frown}{APB})$ (§ 235). $\angle BQC \overset{m}{=} \frac{1}{2}(\overset{\frown}{ARC} + \overset{\frown}{APB})$ (§ 235). Adding, $\angle APB + \angle ARC + \angle BQC \overset{m}{=} \frac{1}{2}(2\overset{\frown}{BQC} + 2\overset{\frown}{ARC} + 2\overset{\frown}{APB})$, or by $\overset{\frown}{BQC} + \overset{\frown}{ARC} + \overset{\frown}{APB}$; that is, by the circle. Hence, the sum of the ∡ named = 4 rt. ∡.

PAGE 152

14. Sum = 6 rt. ∡. Method of proof is the same as that used in Ex. 13.

15. The ⊥ bisector of the line joining the points (§§ 188, 122).

16. Given circle *O*, chord $AB >$ chord CD, and *E* their point of intersection.

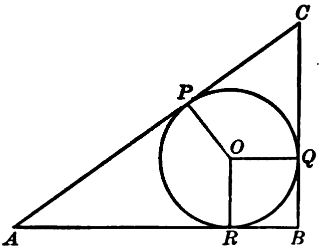


To prove $\angle OED > \angle OEB$.

Proof. On *OE* as a diameter describe a circle intersecting *AB* at *F* and *CG* at *G*. Draw *OF* and *OG*. Then ∡ *OFE* and *OGE* are rt. ∡ (§ 238). $OG > OF$ (§ 208). $\overset{\frown}{OG} > \overset{\frown}{OF}$ (§ 201). $\angle OEG > \angle OEF$ (§ 235).

Converse. Of two unequal chords in a circle, the chord which makes the less angle with the diameter through their point of intersection is the greater. The converse is true.

17. Given rt. $\triangle ABC$ with rt. $\angle$ at *B* with the inscribed circle *O* whose radius is *OQ*.

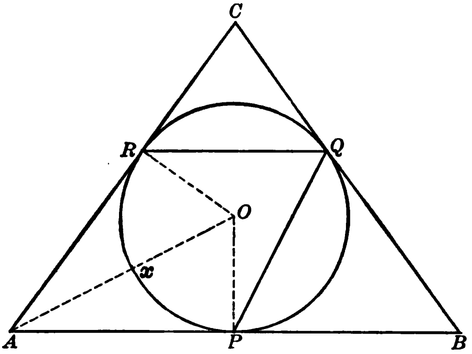


To prove that leg $AB +$ leg $BC = AC + 2\,OQ$.

Proof. Let *P*, *Q*, *R*, be the points of contact, and draw *OP*,

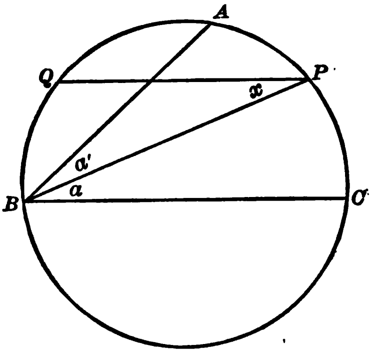
OR, OQ. $AR = AP$ (§ 216), $CQ = CP$ (§ 216) (1). $\triangle$s OQB and ORB are rt. $\triangle$s (§ 210), $\angle B =$ rt. $\angle$ (Hyp.). $\therefore$ $OR \parallel QB$ and $RB \parallel OQ$ (§ 92). $\therefore$ $QB = OR$ and $RB = OQ$ (§ 155) (2). Adding (1) and (2), $AB + BC = AC + 2\,OQ$ (Ax. 2).

18.



Draw also line OA intersecting the circle at x. Then $\angle POx = \angle ROx$ (§ 216, and corr. $\triangle$s in $=$ $\triangle$s). $\therefore$ $\overset{\frown}{Px} = \overset{\frown}{Rx}$ (§ 195). $\angle POx \overset{m}{=} \overset{\frown}{Px}$ (§ 230). $\angle PQR \overset{m}{=} \frac{1}{2}\,\overset{\frown}{RP}$ or $\overset{\frown}{Px}$ (§ 235). $\therefore$ $\angle PQR = \angle POx$ (Ax. 1). But $\angle POA$ is complement of $\angle PAO$ (§ 102). $\angle PAO = \frac{1}{2} \angle RAP$ (§ 216). $\therefore$ $\angle PQR$ is complement of $\frac{1}{2} \angle RAP$ (Ax. 9).

19.



Given PB the bisector of the inscribed $\angle ABC$. PQ a chord $\parallel$ BC.

To prove $AB = PQ$.

Proof. $\angle x = \angle a$ (§ 96). $\angle a = \angle a'$ (Hyp.). $\therefore \angle x = \angle a'$ (Ax. 1). $\overset{\frown}{BQ} = \overset{\frown}{AP}$ (§ 235). Add AQ to each arc. $\therefore \overset{\frown}{AQB} = \overset{\frown}{QAP}$ (Ax. 2). $\therefore$ chord AB = chord PQ (§ 200).

20. Draw OC and AC. Then, in the $\triangle OCP$, CA = radius (Ex. 6, p. 100). $\therefore \triangle OCA$ is equilateral. But $\angle CAO = \angle P + \angle PCA$ (§ 103), or $60° = 2 \angle P$ (§ 82, Ax. 9). $\therefore 30° = \angle P$. $\therefore \angle AEP = 60°$ (§ 106). Similarly by use of $\triangle BOC$, $\angle B = 30°$. $\therefore \angle CDE = 60°$, etc.

21. Draw the radius to the point of contact. Use § 169 and prove that the rt. ▲, in which the lines to be proved equal are the hypotenuses, are equal (§ 79).

1. Construct any two chords of the circular rim of the fragment and then construct the ⊥ bisectors of these chords. The point of intersection of the ⊥s is the center (§ 204).

2. See diagram. Use § 238.

PAGE 153

3. Use § 238.

4. $\angle TPC = \angle PAC + \angle PCA$ (§ 103). But $PA = PC$ (§ 216), etc.

5. Prove $\angle y = \angle O$, and $\angle z = \angle FO'C$.

6. In $\triangle FEH$, $z = y + 180° - x$ (§ 103). $\therefore x - y + z = 180°$. *Ans.*

PAGE 154

7. Use §§ 210, 114.

8. $AP \parallel EP'$, owing to great distance of the sun. $\therefore AP$ (if produced) $\perp QE$ (§ 100). Use § 114. 41° 30′. *Ans.*

9. Latitude of the place is $\overset{\frown}{QZ}$. Hence, if the sun is north of the celestial equator, $\overset{\frown}{QZ} = \overset{\frown}{QS_1} + \overset{\frown}{S_1Z}$. If south, $\overset{\frown}{QZ} = \overset{\frown}{SZ} - \overset{\frown}{QS}$.

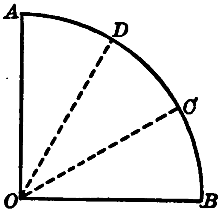
10. At the north pole the latitude = 90°. Hence, QS_1 = sun's elevation above the horizon, which must equal the sun's declination.

11. 6° 7′. At any hour of the day.

12. From the diagram, $\angle HOS = x + x = 2x$.

PAGE 157

3. $112\frac{1}{2}° = 90° + 22\frac{1}{2}°$.



Let AOB be the given rt. ∠, and $\widehat{AB}$ a quadrant with O as a center. With OA as a radius and A and B as centers, describe arcs intersecting AB in D and C. Draw OD and OC. $\angle AOB$ will be trisected by OD and OC. For $\angle DOB = 60°$ (angle of an equilateral △ if chord DB is drawn). $\therefore \angle AOD = 30°$, etc.

5. No.

PAGE 159

4. (1) In diagram on p. 159 make $\angle A = 60°$, $m = 2$ in., $n = 1\frac{3}{4}$ in.
(2) On diagram p. 159, make $\angle A = 60°$, $m = 2$ in., $n = 1$ in.

PAGE 160

3. Use § 238. Hence, since BC is a vertical line AC is ⊥ to it and ∴ horizontal.

PAGE 161

3. Construct an equilateral triangle.

PAGE 162

1. Bisect the exterior angles of the triangle.

2. In first diagram, let A, B, C, D be the vertices of the square, and O the midpoint of AB. A ⊥ from O to AC is the radius of the semicircle.

PAGE 163

2. 81°; 81°; 99°; 99°.

PAGE 165

1. From the center of the given circle draw a ⊥ the given line (§ 129). At the points where this ⊥ intersects the given circle draw tangents to the circle (§ 262). Use §§ 210, 92.

2. From the center of the given circle draw a line || the given line (§ 95). At the points where the line drawn meets the given circle, draw tangents to the circle (§ 262). Use §§ 210, 100.

3. Draw the chord connecting the two given points. Draw the radius ⊥ this chord (§ 129). Through the two given points draw lines || this radius. Use §§ 202, 100, 155, 207.

PAGE 166

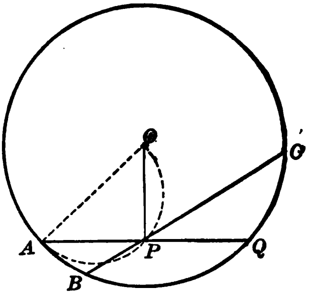
4. At any point in the given line draw a line which makes an angle with the given line equal to the given ∠ (§ 86). Through the given point draw a line || last line drawn (§ 95).

6. Take any point on one of the given || lines as a center, and with a radius equal to the given line-segment describe an arc intersecting the other || line. Connect this point of intersection with the center of the arc. Through the given point draw a line || last line drawn. Use § 157. (Two solutions in general.)

7. Let *O* be the center of the given circle. With any point *P* on the given circle as a center and the given line as a radius describe an arc intersecting the given circle at *Q*. Draw the chord *PQ*. With *O* as a center and the distance of *PQ* from *O* as a radius describe a circle. From *A* draw a tangent to this last circle (§ 264). This tangent produced is the chord required. Use §§ 188, 207. (Two solutions in general.)

8. Draw a concentric circle with the given distance as radius. Use § 264, etc.

9.



Let *O* be the given circle, *A* the given point, and *BC* the given chord. It is required to draw a chord from *A* to be bisected by *BC*.

Draw the radius OA. On OA as a diameter describe the semicircle intersecting BC at P (§ 128). Draw AP and produce it to meet the given circle at Q. Then AQ is the chord required.
For $\angle APO$ is a rt. $\angle$ (§ 238). $\therefore AP = PQ$ (§ 202).
There is no solution when the circle drawn (APO) does not intersect the given chord (BC).

1. (Group 40.) That is, draw the $\perp$ bisector of the line CD (§ 128). Use § 120.
2. Draw the $\perp$ bisector of CD. See Ex. 1. The $\perp$ bisector will meet the circle in two, one, or no points, according to circumstances.
3. Draw the bisectors of the $\measuredangle$ made by the given intersecting lines (§ 84). Use § 125. (Two solutions in general.)
4. With the given point as a center and d as a radius describe an arc intersecting the given line (Post. 3). (Two solutions in general.)

PAGE 167

5. Construct a circle with radius a, and another with radius b, having A and B for their respective centers. In general the two circles will intersect in two points and there will be two solutions. When $a + b =$ line AB, the circles touch and there is one solution. When $a + b < AB$, the circles do not meet and there is no solution.
6. That is, draw the $\perp$ bisector of the line connecting the two given points (§ 128). Also draw two lines $\parallel$ the given line, at the given distance from it (§§ 85, 95). (Two solutions.)
7. § 128, Post. 3.
8. §§ 128, 84.
9. A line $\perp$ the given line at the given point (§ 210).
10. The moving point is always at the distance r from the given point (§ 188). Hence, the locus is the circle having the given fixed point for its center and r for its radius. (See Ex. 6, p. 150, Lemma.)
11. The moving point is equidistant from the two given lines (§§ 210, 188). Hence, the locus is the bisectors of the $\measuredangle$ made by the given intersecting lines (§ 127).
12. The moving point is equidistant from the given $\parallel$ lines (§§ 210, 188). Hence, the locus is a line $\parallel$ the given lines and midway between them.
13. The moving point is always at the distance r from the given line (§§ 210, 188). Hence, the locus is two lines $\parallel$ the given line and at the distance r from it.

14. See Ex. 6, p. 150.

1. Construct any equilateral △. Bisect one of its ∡ (§ 84). Cut off a part of the bisector from the vertex, equal to the given altitude. At the end of the altitude remote from the vertex of the ∠, erect a ⊥ (§ 85), etc.

2. Bisect the base (§ 128). Erect a ⊥ at midpoint of the base = given altitude (§ 85), etc.

PAGE 168

3. At each end of the base and on the same side of the base construct an ∠ = the given ∠ (§ 86).

4. Bisect the given vertex ∠ (§ 84), make the bisector = given altitude, etc.

5. At one end of the given leg construct an ∠ = given acute ∠ (§ 86). At the other end of the leg erect a ⊥ (§ 85), etc.

6. Construct the complement of the given acute ∠, and use Ex. 5.

7. Then with the other end of the altitude as a center, describe arcs with the given sides as radii, etc.

8. Draw a line || one of the given sides and at a distance = the given altitude (§§ 85, 95). With one end of the side thus used as a center, describe an arc with the other side as a radius, etc.

9. Bisect the given ∠ (§ 84) and from the vertex mark off on the bisector a part = the given diagonal. At the other end of the diagonal draw lines || the sides of the given ∠ (§ 95). Use §§ 96, 115, etc.

10. At the end of the given altitude construct an ∠ = complement of the given ∠. At the other end of the altitude erect a ⊥ to the altitude, etc.

11. Bisect the given diagonals and construct a △ with the semi-diagonals as sides and the included ∠ = the given ∠ (§ 253), etc.

12. Use §§ 253, 252.

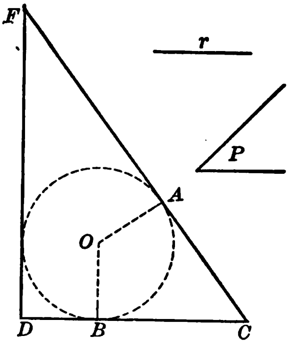
PAGE 169

1. Draw the circumscribed ⊙. In this insert a chord = the given base. Construct the ⊥ bisector of the base (§ 128), etc.

2. Draw the circumscribed ⊙. Insert a chord in this circle = given leg. Draw the diameter from one end of this chord, etc. Use § 238.

3. Draw the circumscribed ⊙ and a diameter in this ⊙. At one end of this diameter construct an ∠ = the given acute ∠ (§ 86), etc. Use § 238.

4.

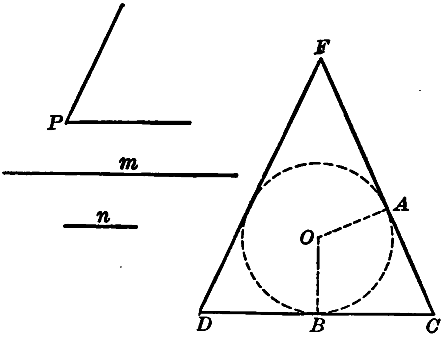


Then, at the ends of the radii of this central ∠ (*A* and *B*) draw tangents to the given circle, meeting at *C*.
Draw *DF* tangent circle and || *OB* (Ex. 1, p. 165) meeting *CA* produced at *F* and *CB* produced at *D*. Then △ *DFC* is the △ required. For *OB* ⊥ *DC* (§ 210). ∴ *FD* ⊥ *DC* (§ 100), etc.

5. Also draw a line || given base at a distance = given altitude (§§ 85, 95). From the points where this || line intersects the circle draw lines to the extremities of the base. (Two solutions in general.)

6. On the given base construct a segment containing the given ∠ (§ 265). With the midpoint of the base as a center and with a radius = given median describe an arc intersecting the circle. From the points of intersection draw lines to the extremities of the given base. (Two solutions in general.)

7.



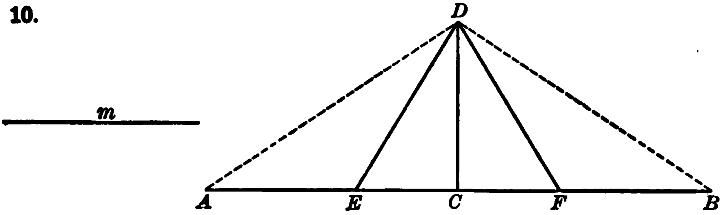
Let *m* be the given side, *P* the given ∠, and *n* the radius of the inscribed ⊙.

Draw the inscribed ⊙. At O construct ∠ AOB supplement of P. At A and B draw tangents to the circle O intersecting at C (§ 262). On CB produced mark off $CD = m$. From D draw DF tangent to circle O (§ 264) and meeting AC produced at F, etc.

8. Draw the circumscribed ⊙. In this circle insert a chord = the given side. At one end of this chord construct an ∠ = the given ∠, etc.

PAGE 170

10.

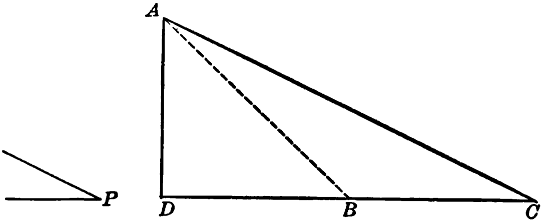


Let m be the given altitude, AB be the given perimeter. At C, its midpoint, construct $CD \perp AB$ and equal to the given altitude m. Connect AD and BD. Construct $\angle ADE = \angle A$, and $\angle FDB = \angle B$. Then EDF is △ required. For $AE = ED$, $FB = DF$ (§ 115), etc.

11. A base ∠ = $\frac{1}{2}(180° -$ vertex ∠). Then follow method of Ex. 9.

12. Also at B construct an ∠ = given acute ∠, etc.

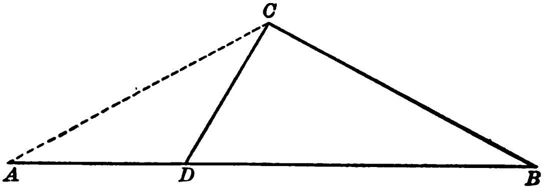
13.



Let BC be the difference of the legs and P the given acute ∠. Produce BC through B and construct $\angle DBA = 45°$. At C construct $\angle ACB =$ given acute $\angle P$. From A, the point where AC and AB intersect, drop $\perp AB$ to DC (§ 129). Then ADC is the △ required. For $\angle DAB = 45°$ (§ 106). $\therefore AD = DB$ (§ 115), etc.

14. Take a line equal to the sum of the legs and at one end of this line construct an ∠ of 45°. With the other end as a center, and the given hypotenuse as a radius, describe an arc cutting the indefinite side of ∠ 45°. From the point of intersection thus formed drop a ⊥ to the line which is the sum of the legs, etc.

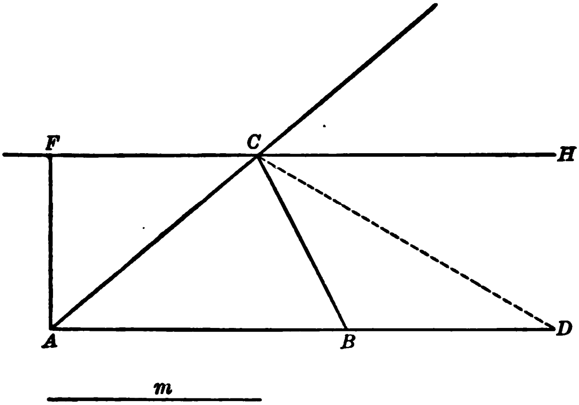
15.



Let x be the given acute ∠ and AB the sum of hypot. and one leg. At B construct $\angle B = \angle x$. At A construct $\angle A = \frac{1}{2}(90° - x)$. At C, the intersection of CA and CB, construct $\angle ACD = \angle A$. Then △ DCB is the △ required. For $\angle CDB = 2 \angle A = 90° - x$ (§ 103), etc.

16. At an end of the line = sum of two sides construct an angle = $\frac{1}{2}$ the given ∠. With the other end as a center and the given side as a radius describe an arc, etc.

17.



Let AD be the sum of the two sides; $\angle CAB$ = given ∠, m = alt. on AB. Then at A draw $AF \perp AD$ and $= m$ (§ 85).

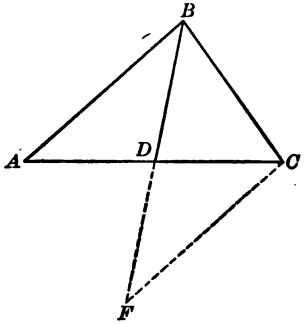
Through F draw $FH \parallel AD$ and intersecting AC in C (§ 95). Draw CD. At C construct $\angle BCD = \angle D$. Then $\triangle ACB$ is the $\triangle$ required, etc.

PAGE 171

1. Construct a right $\triangle$ having the given median for its hypotenuse and the given altitude for a leg (Ex. 12, p. 139). Then with that vertex of this $\triangle$ which is opposite given altitude, as a center, and the median as a radius, describe a circle (use Ex. 6, p. 100), etc.

3. Ex. 12, p. 170. 4. § 252.

5.



Analysis. In the $\triangle ABC$ let AB and BC and the median BD be given. Produce BD to F, making $DF = DB$. Draw FC. Then $\triangle ABD = \triangle FDC$. $\therefore FC = AB$. In $\triangle BCF$ the three sides are known. Hence,

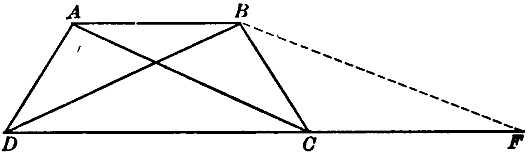
Construction. Construct a triangle (BCF), one of whose sides (BF) is twice given median and whose other sides are the two given sides (§ 252). Draw the median CD and produce it to A, making $AD = DC$. Draw AB. Then $\triangle ABC$ is the required $\triangle$. For $\triangle ADB = \triangle FDC$ (§ 79). $\therefore AB = CF$, etc.

6. Ex. 3, p. 168. The base of the isosceles $\triangle$ = difference of the given trapezoid bases, etc.

7. Ex. 12, p. 139. The leg of the right $\triangle = \frac{1}{2}$ the difference of the given trapezoid bases, added to the shorter base of the trapezoid, etc.

8. § 252.

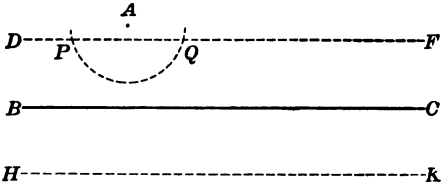
9.



ANALYSIS. Let AB and CD be the bases of the trapezoid $ABCD$. Produce DC to F, making $CF = AB$. Draw BF. Then $ABFC$ is a ▱ (§ 161). $\therefore$ $BF = AC$. Hence,

CONSTRUCTION. Construct the $\triangle$ (DBF) whose base (DF) = sum of bases of required trapezoid, and whose other two sides are its diagonals (§ 252). Through the vertex B draw a line (BA) $\parallel$ base and equal to one base (CF) (§ 95). Then draw $AC \parallel BF$ and meeting DF in C. Draw BC. Then $ABCD$ is the trapezoid required. For $AC = BF$ (§ 157).

PAGE 172



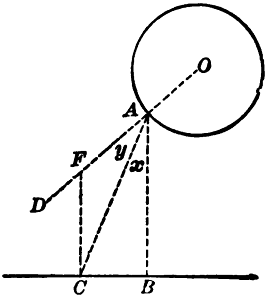
Let A be the given point, BC the given line, r the given radius. Then construct the locus of all points at the distance r from BC, viz., the lines DF and $HK \parallel BC$. Also construct the locus of all points at the distance r from the point A, viz., $\overset{\frown}{PQ}$ (Ex. 6, p. 150, Lemma). These loci may intersect in certain points as P and Q. Then with P and Q as centers describe circles with r as a radius. These circles will be the circles required.

2. Draw lines $\parallel$ the second line at the distance r, etc.
3. Find a point at distance r from each point. (See Ex. 5, p. 167.)
4. Draw a line $\parallel$ the given lines and midway between them. Find a point in this last line at a distance from the given point equal to $\frac{1}{2}$ the distance between the given $\parallel$ lines.
5. Find a point in the given line equidistant from the two given points. (See Ex. 1, p. 166.)

6. Bisect the ∡ made by the intersection of these lines. Use § 126 in the proof.

7. Also draw the ⊥ bisector of AB.

8.



Draw the radius OA and produce it to D. Drop a ⊥ AB from A to the given line BC (§ 129). Bisect the $\angle DAB$ by the line AC (§ 84), meeting BC at C. At C construct $CF \perp BC$ and meeting OD at F. Then F is the center and FC the radius of the required circle. For $\angle x = \angle y$ (Constr.), $\angle x = \angle FCA$ (§ 96). $\therefore \angle y = \angle FCA$ (Ax. 1). $\therefore FA = AC$ (§ 115), etc.

9. Denote the center of the given ⊙ by O. Draw a line $CF \parallel AB$ at a distance = radius of given circle and on the opposite side of AB from the ⊙. Through A draw a line AC ⊥ line CF. Draw OC. At O in OC construct the $\angle COP = \angle ACO$ (§ 86). Let OP meet AC produced at P and intersect the circle at D. Then P is the center of the ⊙ required and PA is its radius. For in $\triangle PCO$, $\angle PCO = \angle O$ (Constr.). $\therefore PC = PO$ (§ 115). But $AC = DO$ (Constr.). $\therefore PA = PD$ (Ax. 3), etc.

1. (Group 45). Bisect the ∡ of intersection of the two given lines (§ 84). From the given point draw ⊥ to the bisectors of these ∡ (§ 129).

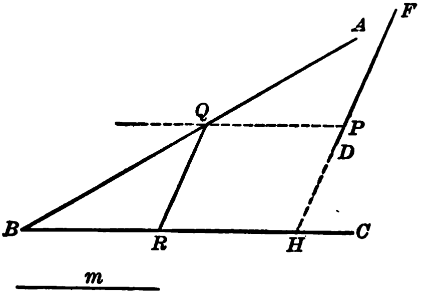
2. See Ex. 12, p. 139.

3. Bisect the ∡ made by the given intersecting lines (§ 84) and produce the bisectors to meet the given circle. Use § 127.

PAGE 173

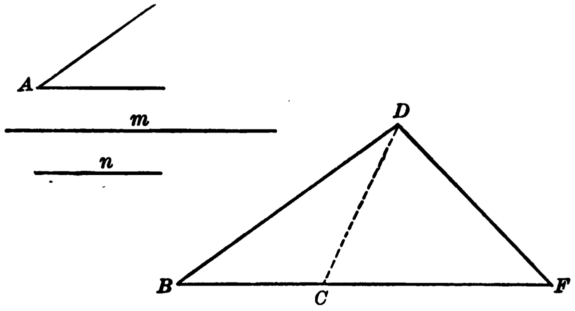
4. Bisect the ∡ made by the given intersecting lines (§ 84). At the given point erect a ⊥ to the line in which the point is and produce it to meet the bisectors, etc.

5.



Let *ABC* be the given ∠, *m* the given line-segment, and *DF* last given line. Produce *DF* to meet *BC* in *H*. On *HF* mark off $HP = m$. Through *P* draw $PQ \parallel BC$ and meeting *BA* in *Q* (§ 95); from *Q* draw $QR \parallel FH$ and meeting *BC* at *R*. Then *QR* is the line required, etc.

6.



Let *m* be the given side, *A* the adjacent ∠, and *n* the difference of the other two sides. Then construct △ *BCD* having $BC = n$, $BD = m$, $\angle B = \angle A$ (§ 253). Produce *BC* to *F* and at *D* in the line *CD* construct $\angle CDF = \angle DCF$ (§ 86). Produce *DF* to meet *CF* at *F*. Then ∠ *BDF* is the △ required. For $CF = DF$ (§ 115). $\therefore BF - DF = BF - CF = BC$.

7. Describe a circle with the given point as a center and the given distance as a radius.

8. Reduce to § 255.

9. Then through the extremity of the last line draw a line ∥ one side of the given ∠ and meeting the other side at the point *O*.

From O draw a line through the given point to meet the other side of the given ∠. The last line drawn is the line required. Use §§ 69, 96, 80.

10. Put the segments of the base together in line to form the base. On this base construct a segment of a circle which shall contain the given vertex ∠ (§ 265). At the point of the base which separates the given segments erect a ⊥ (§ 85) and produce it to meet the arc of the segment, etc.

11. A base ∠ = $\frac{1}{2}(180^\circ -$ vertex ∠$)$.

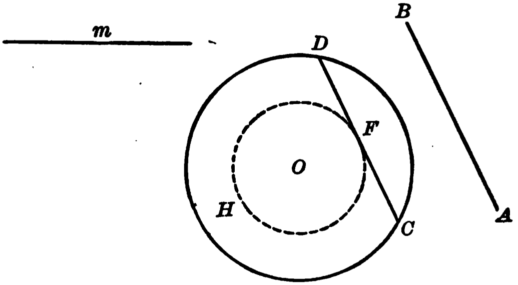
12. From the center of the given circle draw a radius to the given point on its circle. Produce this radius till the produced part = the radius of the required circle. The extremity of the produced part is the center of the required circle.

13. Upon the given hypotenuse as a diameter describe a semicircle. Draw a line ∥ the hypotenuse at a distance = given altitude, etc. Use § 238.

14. Then, with each end of the given base as a center, and one of the given altitudes as a radius, describe an arc cutting the circle. Through each point of intersection and the nearest extremity of the base draw a straight line and produce these lines to meet. Use § 238.

15. At one end of the given altitude construct an ∠ = the complement of one of the given △. At the same point but on the opposite side of the altitude, construct an ∠ = complement of the other given angle. At the other end of the altitude construct a ⊥ to the altitude and produce it to meet the sides of the △ so formed.

16.



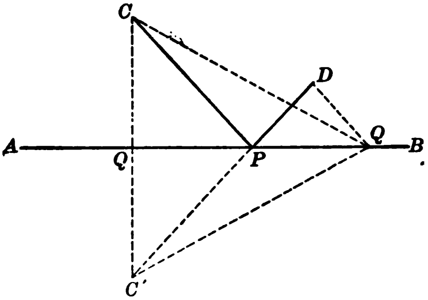
Let O be the given ⊙, m the given line-segment, and AB the given line. Denote the radius of the given circle by R. Bisect the line m. Construct a rt. △ having R for its hypotenuse and $\frac{1}{2}m$ for a leg (see Ex. 12, p. 139). With O as a center, and a radius = the other leg of this rt. △, describe the circle FH. Draw $DC \parallel FH$ and tangent to circle O (Ex. 1, p. 165). Then CD is the chord required. Use §§ 210, 202, 117.

17. Construct the given ∠ and its bisector. Draw a line ∥ one side of the ∠, at a distance from it = given altitude and intersecting the other side of the ∠. Draw a line from this point of intersection through the end of the bisector, etc.

PAGE 174

18. The locus will be a circle having as its diameter the line connecting the given point and the given center.

19.



Draw $CQ \perp AB$ and produce CQ to C', making $QC' = CQ$. Draw $C'D$ cutting AB in P. Draw CP. Then P is the point required.

Proof. In ▲ CPQ and $C'PQ$, $CQ = QC'$ (Constr.), $QP = QP$ (Ident.), $\triangle CQP = \triangle C'QP$ (§ 79). ∴ $\angle CPQ = \angle C'PQ$. But $\angle C'PQ = \angle DPB$ (§ 69). ∴ $\angle CPQ = \angle DPB$ (Ax. 1).

20. Use the same figure and construction as in Ex. 19. Then take any other point in AB except P as Q, and draw DQ and $C'Q$ and CQ. Then in $\triangle C'DQ$, $C'P + PD < C'Q + DQ$ (§ 78). For $C'P$ substitute its equal CP. ∴ $CP + PD < C'Q + DQ$ (Ax. 9). For $C'Q$ substitute its equal CQ. ∴ $CP + PD < CQ + DQ$ (Ax. 9).

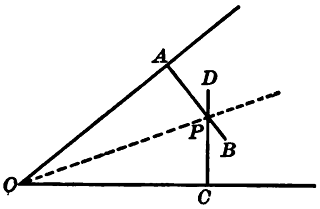
21. Using the center of the larger ⊙ as a center and the difference of the radii of the given circles as a radius, describe a circle. Use §§ 264, 210, 161, 98, 155, 211.

22. Same construction as in Ex. 21, except that the new radius = sum of given radii.

PAGE 175

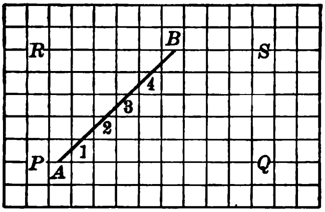
1. Locate two diameters of the circle by use of § 238.

2.



Let O be the given angle. Place the carpenter's square in the two positions, OAB and OCD, OA being equal to OC. Then OP bisects $\angle O$ (§ 117).

3.



On the squared paper let the cross line RS be 5 spaces from the || cross line PQ. Take A, any point on PQ, and with A as a center and a radius = 1½ in., describe an arc cutting RS at B. Denote the points where AB cuts the cross lines || PQ, by 1, 2, 3, 4. Then AB is divided into 5 equal parts at 1, 2, 3, 4 (§ 169). The construction can be made on paper ruled in only one direction.

5. At D construct $\angle BDF = 60°$. Make $DF = BF$. Produce DF to H and construct $\angle KFH = 60°$. Then FK is an extension of AB. To prove this draw BF. In $\triangle BFD$ show $\angle FBD = \angle BFD = 60°$. $\therefore$ BF is an extension of AB and also of KF.

6. Use §§ 96, 80. **7.** See Ex. 12, p. 132.

PAGE 176

9. In $\triangle EAN$, $\angle EAN$ is comp. of $\angle GAE$; $\angle AEN$ is comp. of $\angle EAC$. But $\angle GAE = \angle EAC$ (Constr.). $\therefore$ $\angle EAN = \angle AEN$ (§ 66). $\therefore$ a circle may be described with N as a center and radius AN. Similarly show that H is equidistant from E and C. Through E draw $PQ \perp EH$. Then EP is tangent to arcs AE and EC. $\therefore$ AEC is a compound curve (§ 211). Etc.

10. $KG = KA$ (§ 120). $\therefore$ arc from A passes through G. Also in $\triangle EDG$, $\angle EDG = \angle HAB$, $\angle EGD = \angle HGK$. $\therefore$ $\angle EDG = \angle EGD$ (Ax. 1). $\therefore$ an arc drawn with E as a center and EG as a radius passes through D. Through G draw a line $\perp EK$. This line will be a common tangent to the arcs NG and GD (§ 211). Hence, these arcs form a compound curve.

PAGE 177

1. §§ 183, 196, 197, 200, 201.
2. §§ 184, 194, 198, 201, 202, 207, 208, 209.
3. §§ 182, 192, 202.
4. §§ 195, 197, 229, 230.
5. §§ 233, 234, 235, 237, 238, 239.
6. §§ 185, 210, 211, 212, 213, 216, 218, 242.
7. Three; two; four; three if one is at the vertex, otherwise four.
8. One; one; three.
9. See §§ 258 and 261.
10. §§ 238–239.
11. If we know that in the same circle or in equal circles two chords are equal, Prop. IX enables us to determine, without effort, that the chords are equidistant from the center, and conversely.
12. If we know that a straight line is tangent to a circle, Prop. XII enables us to determine without effort that it is $\perp$ the radius drawn to the point of contact.
13. If we know the length of one of the tangents drawn from a given point to a given circle, we are enabled to determine the length of the other tangent thus drawn, without measuring it.
14. If in a circle or in two equal circles we know the relative size of two arcs intercepted by two central angles, Prop. XVI enables us to determine without effort the relative size of these angles.

18. See § 251.
19. The number of degrees of angle in $\angle ABC$ equals the number of degrees of arc in $\overset{\frown}{AC}$.

PAGE 180

1. $5 : x = x : a.$

2. 10.

3. 15.

4. $\frac{pq}{a}.$

5. 2.4

6. .54.

7. $12\,b.$

8. 96.

9. 9; $\frac{1}{4}.$

10. $3\sqrt{2}$; .08.

11. $8\frac{1}{3}$; $\frac{1}{18}.$

12. $9\,p^2.$

13. $\sqrt{a^2 - b^2}.$

PAGE 181

1. $a : p = q : b$; $b : p = q : a$, etc.

2. $x : 3 = 2 : x + 1$; $3 : x = x : 5.$

3. $x : y = 4 : 3.$

4 $\frac{7}{5}.$

5. $\frac{b}{a}.$

6. $\frac{p + q}{a + b}.$

7. $\frac{b - d}{a - c}.$

PAGE 182

1. $x : b = a : c.$

2. $a : x = c : b.$

3. $c : a = b : x.$

4. $c : a = b : x.$
$2c : a = b : x.$

5. $1 : x = x : 3.$
$a + b : x = x : a - b.$

1. (§ 286). $15 : 3 = 10 : 2.$

2. $2x : 5 = 3x : 7.$

3. $\sqrt{x + 3} : 5 = \sqrt{2x - 1} : 7.$

4. It simplifies the proportion by the cancellation of like terms.

PAGE 183

1. $9 : 3 = 6 : 2.$

2. $2x : 5 = 3x : 7.$

3. $7x : \sqrt{3x - 5} = 4x : \sqrt{2x + 1}.$

4. It simplifies the proportion by the cancellation of like terms.

6. Use § 287. $3x : 2y = 8 : 2.$

PAGE 184

1. $6x : 4y = 12 : 8.$

2. $2\sqrt{x + 7} : 2\sqrt{5} = 4x : 2.$

3. See Ex. 4, p. 183.

1. (§ 289). $24 : 6 = 4 : 1.$

2. $\frac{x + 6}{y + 6} = \frac{x + 1}{y + 3}.$

3. $\frac{x}{y} = \frac{2}{5}.$

PAGE 185

1. $\frac{6}{7}$.
2. $\frac{3}{5}$.

1. (§ 291). 8 : 27.
2. 8 : 125.
3. 4 : 49.
4. $\frac{9}{16}$.
5. $\frac{15}{2}$.
6. (1) When we know the value of the ratio of like roots of two quantities, § 291 enables us to find the ratio of the numbers themselves.

   (2) It enables us often to find the value of an unknown quantity involved in a proportion containing radicals.

PAGE 186

1. $6x$.
2. $\frac{(a-b)^2}{a+b}$.
3. 12.
4. Use § 278.
5. $2x : 2 = 12 : 2;\ x = 6$.
6. $bqx = bpy$ (§ 278).
   $\therefore\ qx = py$ (Ax. 5).
   $\therefore\ x : y = p : q$ (§ 281).
7. $P : W = l : L$.
8. 40 lb.
9. Any three of the four quantities P, W, l, L being given, it enables us to find the remaining one.

11. $\frac{2x^2}{4x^2} = \frac{6 + 2\sqrt{1+x}}{8 - 4\sqrt{1-x}}$ (§ 284); hence, $\frac{1}{1} = \frac{3 + \sqrt{1+x}}{2 - \sqrt{1-x}}$ (§ 227).

12. $\frac{2x^3 - 6x^2}{10x - 14} = \frac{4x^3 + 8x^2}{-6x + 8}$ (§ 288); hence, $\frac{x-3}{5x-7} = \frac{2x+4}{-3x+4}$ (§§ 284, 227).

13. $\frac{x+2}{x-2} = \frac{3x+5}{3x-8}$ (§ 291); $\frac{x}{2} = \frac{6x-3}{13}$ (§§ 288, 227).

PAGE 188

1. 9; 6.
2. 5.
3. $\frac{ac}{b}$.
4. $4\frac{1}{6}$; $5\frac{1}{7}$; $2\frac{3}{35}$.

4.5 7.5 3

PAGE 189

3. Use the method of § 297, making $p = n$.
4. $\therefore\ a + b : a - b = c : x$ (§ 288). Use § 297.

PAGE 190

3. In general use the method of § 298, but on AP lay off the three lines m, n, p in succession.

PAGE 191

1. Yes.
2. No.
3. Yes.

PAGE 192

1. 8.
2. 1.5.
3. 6; 8.
4. 14; 7.
5. 5; 15.
6. 4; 6.

PAGE 194

1. Yes.
2. No, for $\frac{4}{10}$ does not equal $\frac{6}{12}$.
3. 32.9; 252; 23.1; 16.8.

PAGE 195

1. Yes, when they are mutually equiangular; no.

3. Use § 304. Also let *BD* and *CA* intersect in the point *O*. Then the $\triangle$ *BOA* and *COD* are similar.

4. 8 into 6 and 2; 10 into 7½ and 2½.

6. § 83; § 89 (twice).

PAGE 196

1. No.

2. Because it is not given that $\angle$ *A* and *A′* are equal.

3. Yes.

PAGE 197

1. Use §§ 238, 305.
2. Use §§ 96, 304.

3. $\triangle$ *APB*, *FHR*, *PQR* are similar (§§ 108, 305, 306).

4. § 305.

5. The $\triangle$ are *ABQ*, *CPQ*, *APD*. Use § 305.

6. Draw *DB* and *AC* on the figure, p. 208 (text-book). Two pairs.

7. See Ex. 14, p. 145.
8. Use §§ 300, 97, 304, 302.

9. 372 yd. It enables us to determine the distance from a given place to an inaccessible object.

10. Use §§ 97, 305.

PAGE 198

1. 5.6; 7.

PAGE 199

1. Similarly $\frac{AH}{AC} = \frac{1}{3}$. Use Ax. 1 and § 310.

2. Let ABC and $A'B'C'$ be the given similar $\triangle$ and BD and $B'D'$ the corresponding medians. Then in the $\triangle$ ABD and $A'B'D'$, $\angle A = \angle A'$ (§ 302). Also $AB : A'B' = AC : A'C'$ (§ 302). But $AC : A'C' = AD : A'D'$ (§ 227). $\therefore$ $AB : A'B' = AD : A'D'$ (Ax. 1). $\therefore$ the $\triangle$ ABD and $A'B'D'$ are similar (§ 310). In like manner the $\triangle$ BDC and $B'D'C'$ may be proved similar.

3. Use §§ 97, 305.

4. In $\triangle$ ABP and $A'B'P'$, $A'B' \parallel AB$ (§ 300). Prove $\triangle$ ABP and $A'B'P$ similar by § 305. $\therefore$ $AB : A'B' = BP : B'P$. Similarly, $BC : B'C' = BP : B'P$. $\therefore$ $AB : A'B' = BC : B'C'$, etc. Use § 309.

5. In Ex. 4, change "within" to "without." Use the same method of proof as in Ex. 4.

6. Let P fall on the side AC. Prove $AB \parallel A'B'$; $BC \parallel B'C'$. Also, $\frac{AP}{A'P} = \frac{AB}{A'B'} = \frac{BP}{B'P} = \frac{BC}{B'C'} = \frac{PC}{PC'}$. $\therefore$ $\frac{AP}{A'P} = \frac{PC}{PC'}$, whence $\frac{AP}{PC} = \frac{A'P}{PC'}$ (§ 284). $\therefore$ $\frac{AP + PC}{PC} = \frac{A'P + PC'}{PC'}$ (§ 286). Or, $\frac{AC}{PC} = \frac{A'C'}{PC'}$. Hence, $\frac{AC}{A'C'} = \frac{PC}{PC'} = \frac{BC}{B'C'}$. Use § 309.

7. 110 ft.

8. 1085 ft.

9. By § 301, $8 + 10 : BC = 9 : 15$. $\therefore$ $BC = 30$. *Ans.*

10. Let ABC and $A'B'C'$ be the two similar triangles. Let $AB = BC$. Then $AB : BC = A'B' : B'C'$ (§ 302). $\therefore$ $A'B' = B'C'$ (§§ 284, 282).

PAGE 203

2. Lengths of lines drawn are 1½ in.; 2½ in.; 6 in.

3. 2½ in. by 1¼ in.

4. 100 ft.

5. Facilitates (1) a more ready comparison of parts; (2) discussion of changes; (3) sending information to a distance.

PAGE 204

1. 140 rd.

2. Twice.

PAGE 205

1. Divide the squares into triangles and use §§ 310, 313.

2. $137\frac{1}{2}$ miles.

3. Distance $= 1{,}000{,}000 \times \frac{11}{4}$ in. $= 43.40 +$ mi. *Ans.*

4. Draw a pair of corresponding diagonals denoted by h and h'. Then, by § 312, $h : h' = a : a'$. $\therefore P : P' = h : h'$ (Ax. 1).

5. See Ex. 1, p. 199; 680 yd.

6. Prove $\frac{PA}{PA'} = \frac{AB}{A'B'} = \frac{PB}{PB'} = \frac{BC}{B'C'} = \frac{PC}{PC'}$. $\therefore \frac{PA}{PA'} = \frac{PC}{PC'}$ (Ax. 1). Use § 299.

7. $\frac{OB}{OA} = \frac{11}{3} = \frac{OD}{OC}$ (Hyp.). Use § 310.

8. 10.

9. 15; $7\frac{1}{2}$.

10. Prove $\triangle AKM = \triangle AMC$ (§ 80). $\therefore KM = MC$. $\therefore DM \parallel BK$ (§ 300). $AK = AC$ (corr. sides of $=$ $\triangle$s). $\therefore 7 - BK = 4.6$. $\therefore BK = 2.4$, $MD = 1.2$. *Ans.*

PAGE 207

1. 6; $2\sqrt{13}$; $3\sqrt{13}$.

2. .9; .7; $\sqrt{.63} = \frac{3}{10}\sqrt{7} = .793 +$.

3. First find $FC = .32$; then $BC = \frac{4}{25}\sqrt{5} = .357 +$.

PAGE 208

2. Construct the mean proportional between 1 and 2; between 2 and 3.

3. Construct the mean proportional between 1 in. and 5 in.

PAGE 209

1. 6.

2. 12, 6.

3. $x(11 - x) = 6 \times 4$. $\therefore$ 8, 3. *Ans.*

4. $\frac{ab}{c}$.

5. $AF = \frac{r}{2} \pm \sqrt{\frac{r^2}{4} - pq}$. The ± sign means that two constructions of the figure are possible according as AF is greater or less than FB.

PAGE 210

1. 6.

2. $4\overline{AF}^2 = 144$. $\therefore AC = 24$. *Ans.*

3. .16.

4. Let OA cut the circle in D. Then $OD = 21 - 15 = 6$. $\therefore AC \times AF = 36 \times 6 = 216$ (§ 325).

1. (Group 51.) $\triangle$ ADC and BFC are rt. $\triangle$ having the acute $\angle C$ in common. Use § 305.

2. $\triangle$ AOF and ACD are rt. $\triangle$ having the acute $\angle CAD$ in common. Use §§ 305, 306.

3. Denote a base $\angle$ of each by A. Then the vertex $\angle$ of each $\triangle = 180° - 2\angle A$. Hence the $\triangle$ are mutually equiangular. Use § 304.

4. Use figure Ex. 10, p. 212 (text-book). Denote the point where AC and BD intersect by F. Then $\triangle$ AFD and BFC are mutually equiangular (§ 96), and $\therefore$ similar (§ 304).

5. $\angle P$ is measured by $\frac{1}{2}$ arc AC (§ 235). Also $\angle FAC$ by $\frac{1}{2}$ arc BC (§ 235). $\therefore \angle P = \angle FAC$. Also $\angle C = \angle C$ (Ident.). $\therefore$ $\triangle$ APC and AFC are similar (§ 305).

6. $\angle AEB$ is a rt. $\angle$ (§ 238). $\angle ABD$ is a rt. $\angle$ (§ 210). Use § 305.

7. $\angle APQ$ is a rt. $\angle$ (§ 238). Use § 305.

PAGE 211

8. $\angle ABD = \angle APC$ (each $\overset{m}{=} \frac{1}{2}\overset{\frown}{AC}$, § 235). Also, $\angle BAD = \angle PAC$ (Hyp.). Use § 305.

9. Draw a pair of corresponding diagonals in the given rectangles. Then each pair of corresponding $\triangle$ in the two rectangles are similar (§ 310). Hence, the rectangles are similar (§ 313).

10. In the $\odot$ CAB, $\angle BAD \overset{m}{=} \frac{1}{2}\overset{\frown}{AB}$ (§ 241). Also $\angle C \overset{m}{=} \frac{1}{2}\overset{\frown}{AB}$ (§ 235). $\therefore \angle BAD = \angle C$. In like manner, $\angle CAB = \angle D$. $\therefore$ the $\triangle$ are similar (§ 305).

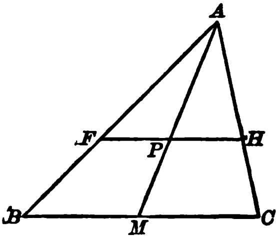
1. In the similar $\triangle$ ACD and BFC, $AD : BF = AC : BC$ (§ 302). $\therefore AD \times BC = BF \times AC$ (§ 278), etc.

2. By Ex. 5, p. 210, $\triangle$ APC and AFC are similar, etc. $CP \times CF = \overline{CA}^2$ (§ 278). CA is constant. $\therefore CP \times CF$ is constant.

3. See Ex. 4, p. 210.

4. ▲ *ABC* and *PBC* are similar by Ex. 3, p. 210, etc.

5.



Let *AM* be a median of △ *ABC*, *FH* ‖ *BC*. Let *FH* intersect *AM* in *P*. Then by means of similar ▲ show that $\frac{BM}{FP} = \frac{AM}{AP} = \frac{MC}{PH}$. ∴ *FP* = *PH* (§ 282).

PAGE 212

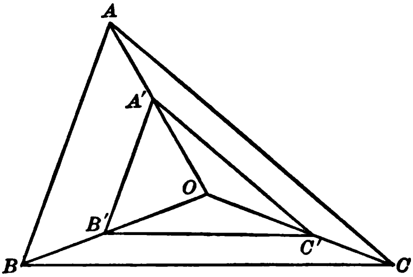
6. The ▲ *APF* and *FQB* are similar (§ 305).
∴ *PF* : *FQ* = *AF* : *FB* (§ 302). But $\frac{AF}{FB}$ is constant, etc.

7. ∠ *ABF* = ∠ *FBC* (measured by ½ of equal arcs (§ 235)). ∴ in the △ *ABC*, *AB* : *BC* = *AE* : *EC* (§ 301)

9. Use § 324.

10. $\frac{PQ}{BC} = \frac{AP}{AB} = \frac{DT}{DC} = \frac{RT}{BC}$ (§§ 294, 295). Use § 282.

11.



▲ *ABO* and *A′B′O* are similar (§ 305). ▲ *OB′C′* and *OBC* are similar (§ 305). ∴ *A′B′* : *AB* = (*OB′* : *OB*) = *B′C′* : *BC* (§ 302).

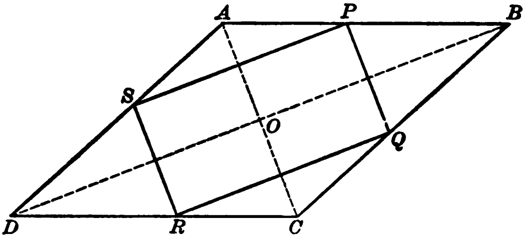
But $\angle A'B'C' = \angle ABC$ (§ 112). ∴ △ $A'B'C'$ and ABC are similar (§ 310).

12. $\frac{AR}{RQ} = \left(\frac{BR}{RD}\right) = \frac{RP}{AR}$ (§ 302). ∴ $\overline{AR}^2 = RQ \times RP$ (§ 278).

1. Then use §§ 101, 146.

2. Use Ex. 8, p. 93, and Ex. 1, p. 212.

3.



Let $ABCD$ be the given rhombus, and P, Q, R, S be the midpoints of its sides. Then $PQRS$ is a ▱ (Ex. 1, p. 212). But $AC \perp DB$ (§ 121), $SP \parallel DB$, $PQ \parallel AC$. ∴ $\angle SPQ = \angle AOB$ (§ 112). In like manner $\angle PQR$ = a rt. $\angle = \angle QRS = \angle RSP$. ∴ $PQRS$ is a rectangle (§ 149).

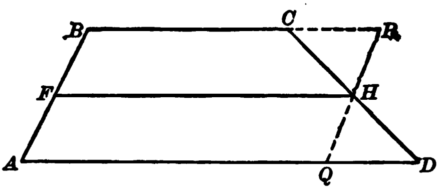
4. Use § 305.

PAGE 213

5. Draw the altitude AH. Then the △ AHC and BFC are similar (§ 305). ∴ $AC : HC = BC : FC$ (§ 302). But $HC = \frac{1}{2} BC$ (§ 118). ∴ $AC : \frac{1}{2} BC = BC : FC$ (Ax. 9). ∴ $AC \times FC = \frac{1}{2} \overline{BC}^2$ (§ 278). Or, $2\, AC \times FC = \overline{BC}^2$ (Ax. 4).

6. Draw the chord BD and prove the △ ABE and ABD similar (Ex. 5, p. 210).

7.



Let $ABCD$ be the trapezoid and F and H the midpoints of its legs AB and CD. Then $FH \parallel AD$ and BC, otherwise § 296 (last sentence) would be violated. Through H draw $PQ \parallel AB$ and meeting

AD in Q and BC produced at P. **Prove** $\triangle CHP = \triangle HQD$ (§ 80). $\therefore CP = QD$. $FH = AQ$ (§ 155) $= AD - QD$. $FH = BP$ (§ 155) $= BC + PC$. $\therefore 2FH = AD + BC$, etc.

8. Denote the tangent by RTS. Then $\angle P = \angle QTR = \angle STQ' = \angle P'$ (§§ 235, 241), etc.
9. Let the point P lie between Q and R. $\angle RQP = \angle RTP = \angle P'TR' = \angle P'Q'R'$ (§§ 235, 241), etc.
10. Draw the diameter AR intersecting CD in F. ▲ AFQ and ARP are similar (§ 305). $\therefore AR : AP = AQ : AF$. $\therefore AP \times AQ = AR \times AF$ (constant), etc.
11. In the similar ▲ ABF and DBC, $AB : AF = BD : CD$. $\therefore AB \times CD = AF \times BD$. In sim. ▲ BCF and ABD, $BC : FC = BD : AD$. $\therefore BC \times AD = CF \times BD$. Adding, $AB \times CD + BC \times AD = (AF + FC) BD = AC \times BD$.

1. (Group 54). In the similar ▲ ABF and BFC, $AB : AF = BC : BF$, etc.
2. $\overline{AB}^2 = AC \times AF$, and $\overline{BC}^2 = AC \times FC$ (§ 319), etc.
3. Draw the chord QB. Then ▲ AQB and APB are similar (§ 305), etc.
4. See Ex. 2, p. 199. **5.** Use §§ 69, 238, 304. **6.** Use §§ 96, 235, 304.

PAGE 214

7. $\angle AFB$ is a rt. $\angle$ (§ 238). **Prove** ▲ DAB and ABC each similar to $\triangle AFB$ (§ 305), hence to each other (§ 306), etc.
9. § 323.
10. Draw the line of centers and the radii to the points of contact. **Prove** the ▲ thus formed similar (§§ 69, 82, 305), etc.
11. The line joining the vertex to the midpoint of the base (See Ex. 5, p. 211).
12. Denote the center by O. **Prove** $\angle POR$ a rt. $\angle$. (For $\angle POA = \angle POQ$, etc.) Use § 319.
13. $OA \times OB = \overline{OP}^2$ (Hyp.). $\therefore$ ▲ OPA and OPB are similar (§ 310).
14. By sim. ▲, $HP : FP = CH : FB = CH : AF = CK : AK = HK : FK$.

1. Construct a fourth proportional to c, a, b; to $2\,c, a, b$ (§ 297).
2. § 321. Construct a mean proportional between $a + b$ and $a - b$.
3. § 321. Construct a mean proportional between $3\,a$ and b.

4. § 321; §§ 321, 128. Construct a mean proportional between 1 and 3. To construct $\frac{1}{2}\sqrt{5}$, construct a mean proportional between 1 and 5 and bisect the result.

PAGE 215

5. *I.e.*, proportional to 12, 3, 4 (§ 298).

6. § 301. That is, bisect the ∠ of the △ opposite the side to be divided.

7. §§ 321, 298. That is, find a mean proportional between 1 and 2 (§ 321). Then use § 298.

8. These ▲ are = (§ 80). ∴ to construct, draw $PL \parallel AC$ and meeting BC at L, bisect CL, etc.

9. $PQ = QR$. ∴ $OQ \perp PR$ (§ 203). ∴ to construct PQ, on OP as a diameter construct a semicircle, etc. Use § 238.

10. Use § 323.

11. They are equal (§ 301). ∴ divide the chord AB into line segments which shall be as 2 : 3 (§ 298) and on AQ construct a segment of a ⊙ which shall contain an ∠ = $\frac{1}{2}$ ∠ inscribed in segment APB (§ 265).

12. See Ex. 9, p. 169.

13. § 316.

14. Sides of required △ = chords subtended by inscribed ∡ = ∡ of given △.

15. Suppose the construction made and lines drawn from the center to the points of contact. Then ∡ at the center = supplements of ∡ of given △. Hence to construct, at the center of the given ⊙ construct adj. ∡ = supplements of the ∡ of the given △. At the points where the radii which separate these ∡ meet the circle draw tangents to the circle.

16. Draw a line bisecting two opposite sides of the rectangle. Or let $ABCD$ be the given rectangle. On AD, its lower base, describe a semicircle intersecting BC in F. Draw $FH \perp AD$. Then FH divides the rectangle as required. For $AH : HF = HF : HD$ (§ 320). Use Ex. 9, p. 211. In case the semicircle does not reach the side BC, the latter method of construction cannot be used.

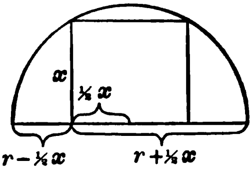
PAGE 216

17. By similar ▲, $AH : DE = (AB : DB) = AK : DG$. But $DE = DG$. ∴ $AH = AK$ (§ 282). Hence, Construction. Draw AK the altitude of the given △. From A draw $AH \parallel BC$ and $= AK$. Draw HB intersecting AC in E. Draw $ED \parallel BC$ and meeting AB in D. Draw DG and $EF \perp BC$, etc.

1. Use the method of the Ex. in § 327.

2. Hence, $x = -a \pm a\sqrt{2}$. Construct $a\sqrt{2}$, then $a\sqrt{2} - a$; use Ex. 12, p. 139.

3.



Denote a side of the required square by x. Then $r - \frac{1}{2}x : x = x : r + \frac{1}{2}x$ (§320). ∴ $x^2 = r^2 - \frac{1}{4}x^2$, or $x^2 = \frac{4r^2}{5}$, or $x = \frac{2}{5}\sqrt{5r^2}$. ∴ to construct x, construct a mean proportional between r and $5r$ and take $\frac{2}{5}$ of the result.

4. Denote the first given line by a, the other given line by b, and the required part by x. Then $a - x : x = x : b$.

$$\therefore\ x = \frac{-b \pm \sqrt{b^2 + 4ab}}{2}.$$

Hence, construct a mean proportional between b and $b + 4a$, etc.

PAGE 217

1. Divide the given line AB in the ratio of $m : n$ (§ 298).

2.

Let d be the difference of the two required lines and m and n two other lines in the given ratio. Denote the smaller of the required lines by x. Then the other line required is $d + x$. Hence $m : n = d + x : x$ (Hyp.) or $m - n : n = d : x$ (§ 287). Hence construct a 4th proportional to $m - n$, n, and d, etc.

3. Use the figure of Ex. 10, p. 212 (text-book). Let $ABCD$ be the given trapezoid and PT the line $\parallel$ the base BC which divides $ABCD$ into two similar trapezoids $APTD$ and $PBCT$. Then $AD : PT = PT : BC$ (§ 302). Hence,

Construction. Construct the mean proportional between AD and BC (§ 321). From any point X in AB draw a line $XY \parallel AD$ and = this mean proportional. Through Y draw a line $\parallel AB$ and meeting CD in T. From T draw $TP \parallel BC$, etc.

Proof. The trapezoids $APTD$ and $PBCT$ are mutually equiangular (§ 97). Let AB and CD be produced and meet at O. Then $OA : OP = (AD : PT = PT : BC) = OP : OB$. $\therefore OA - OP : OA = OP - OB : OP$. $\therefore AP : OA = BP : OP$. $\therefore AP : BP = (OA : OP) = AD : PT$, etc.

4. Let AB be the smaller given line. Produce it through B to C, making AC = larger given line. Through C and B pass any convenient circle. From A draw a tangent to ⊙ (§ 264), viz. AD. Then $\overline{AD}^2 = AB \times AC$ (§ 324).

5. Construct by drawing a line from one point A through the other, B, to meet given line at P. Find the mean proportional between PA and PB (§ 321). On the given line lay off PC = this mean proportional. Through A, B, and C pass a circle (§ 256). (Another solution is obtained by laying off PC' in the opposite direction from C.)

6. Denote the external segment by x. From the given point draw a tangent (= a). Then $2x : a = a : x$ (§ 324). Use § 327.

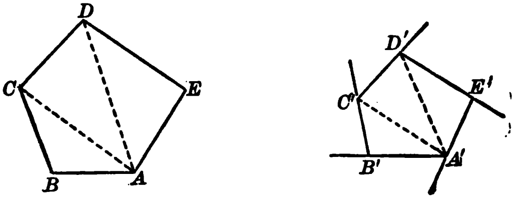
7. $m^2x^2 = \frac{mt^2}{m+n}$. $\therefore t : mx = mx : \frac{mt}{m+n}$.

Given the lines m, n, t, find $\frac{mt}{m+n}$ by constructing the 4th proportional to $m+n, m, t$. Find mx by § 321.

8. From the point of division so made draw a line to P. Through P draw a ⊥ to the last line, etc. Use § 295.

9. Through the points of division on PQ and QR draw a line, etc. Use §§ 305, 302.

10.



Given the polygon $ABCDE$ and the line $A'B'$.

To construct on $A'B'$ a polygon similar to $ABCDE$ and similarly placed.

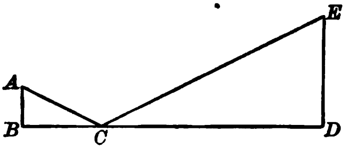
CONSTRUCTION. In the given polygon, draw the diagonals AC and AD, dividing the polygon into triangles. At B', on the line $A'B'$, construct $\angle A'B'C'$ equal to $\angle B$; and at A' construct $\angle B'A'C'$ equal to $\angle BAC$ (§ 86). Produce the lines $B'C'$ and $A'C'$ to meet at C'. $\therefore \triangle ABC \sim \triangle A'B'C'$ (§ 305). Proceed in like manner, using § 305 twice again, and § 313.

11. Divide the given line into parts in the given ratio by § 295. Upon the given line as a diameter construct a circle. Use § 265. Or, construct a diameter $\perp$ given line, and a chord from end of second diameter through the point of division of the first, etc.

PAGE 218

1. AB, BC, AD, so as to be able to use the proportion $AB : BC = AD : DE$.

2.



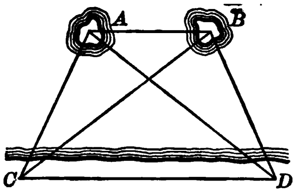
Let AB be the observer, ED the tree, and let the mirror be at C. Then $\angle ACB = \angle ECD$ (Ex. 2, p. 105). $\therefore$ the $\triangle$ are similar (§ 305). $\therefore BC : AB = CD : ED$, or $6 : 5\frac{1}{2} = 120 : ED$. $\therefore ED = 110$ ft. *Ans.*

3. $\angle FBC = \angle EAB$ (§ 114). Denote the height of the tree by x. $\therefore x : 150 = 4\frac{1}{2} : 6$. $\therefore x = 112\frac{1}{2}$ ft. *Ans.*

4. See Ex. 9, p. 197 (text-book). Construct the diagram to scale and on it measure AB, and hence, by use of a proportion, find the distance represented by AB.

PAGE 219

5.



Let C and D be accessible, A and B inaccessible. Measure CD and the angles ACD, BCD, ADC, BDC. Then construct a drawing to scale, and on the drawing measure AB, and then compute the distance represented by AB.

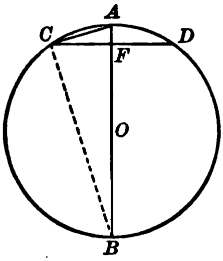
6. $\therefore \frac{\overline{FB}^2}{\overline{FA}^2} = \frac{1}{2}$. $\therefore \frac{FB}{FA} = \frac{1}{\sqrt{2}} = \frac{1}{1.41+} = \frac{5}{7}$ approx.

7. $\therefore$ $SR = 60$. $\therefore$ 60 lb. *Ans.*

8. The diagram of Ex. 7 would need to be changed so that $BC = 18$, and $AC = 100$. $\therefore AB = \sqrt{\overline{100}^2 + \overline{18}^2} = 101.6 +$ ft. $\therefore 101.6 : 18 = 1800$ lb. $: x$. $\therefore x = 318.8 +$ lb. *Ans.*

PAGE 220

11.



Let O be the center of the earth, and CAD be a circle through the tops of the three stakes. Then AB is $\perp$ bisector of the chord CD (§ 205). $\therefore$ by § 320, $AF : AC = AC : AB$. $\therefore AF = \frac{\overline{AC}^2}{AB} = \frac{4\overline{AC}^2}{4AB} = \frac{(2AC)^2}{4\text{ (diam. of earth)}}$. Hence, AF varies as the square of $2AC$. But for a small arc, $2AC$ approximately = arc CAD. Bulge for 4 miles $= 2^2(\frac{2}{3}$ ft.) $= 2\frac{2}{3}$ ft. *Ans.*
Similarly, $10\frac{2}{3}$ ft.; $42\frac{2}{3}$ ft.; $170\frac{2}{3}$ ft. *Ans.*

12. Denote the distance in miles by x. Then $x^2 : 2^2 = 11 : \frac{2}{3}$. $\therefore x = 8 +$ miles. *Ans.*

13. 16 mi.+.

14. 209.75.

15. 44; 91; 687 −; 1418 −.

PAGE 221

16. Let the $\perp$ from B meet AD at F, and that from D meet AC at H. Then $AF : 6000 = 1 : 1.305$. $\therefore AF = 4597 +$. $BF : 6000 = 1 : 1.556$. $\therefore BF = 3856 +$. $3856 + : FD = 1 : .839$. $\therefore FD = 3235 +$, etc. $CD = 4166 +$. *Ans.*

PAGE 222

5. Not unless the polygons have three sides.

6. Not unless the polygons have three sides.

7. §§ 320, 322.
8. §§ 324, 325.

9. §§ 302, 312, 313, 317, 318.

10. §§ 293, 294, 296, 299, 300, 301, 302, 304, 305, 306, 309, 310, 311, 316, 319.

11. Each line on the object is fifty times as long as the corresponding line on the drawing; 1200 times as long.

12. Two.
14. Three.

PAGE 224

3. 4.
5. 24.
7. 32.
9. 27.
4. 24.
6. 13½.
8. 27.
10. 60.

PAGE 225

17. 13½ sq. yd.; 13½ sq. mi.; 1350 sq. mi.

PAGE 227

1. 16 : 3.
2. 1944.
3. 75 : 256.
4. 8⅓ ft.

PAGE 228

1. 21⅞ yd.
2. 17.5.
3. 9 sq. in.

PAGE 230

1. 240 sq. in.
2. 20.3 ft.

PAGE 231

1. 75.917 sq. in.
2. 2 ft.

PAGE 232

1. 1½ sq. ft.
2. 16 in.; 32 in.

1. (Group 61). 35 sq. in.; 35 sq. in.

2. Area of $\triangle PQT = \frac{1}{2}PT \times h = \frac{1}{2}TR \times h =$ area $\triangle TQR$ (§ 343, Ax. 9, § 343).

3. Use §§ 159, 343, Ax. 1. **4.** 1 ft. 4 in. **5.** 4.8 ft.

6. In the world about us a vastly larger number of rectangles occur than of equilateral triangles; and it is easier to resolve rectangles into squares than into equilateral triangles.

7. $831\frac{1}{3}$ sq. ft.; 1360 sq. ft.; 1275 sq. ft.

PAGE 234

1. 962 sq. in. **3.** 2.24 ft. **5.** 168 sq. ft.

2. 9. **4.** $\frac{2K - b'h}{h}$. **6.** The base and altitude.

7. The base and altitude, or the three sides. See Formula 5, under *Areas*, p. 294.

8. The two bases and the altitude.

9. Yes. **11.** No. **12.** 447 sq. in. **13.** $\frac{8^2 \times 4}{4^2 \times 8} = \frac{8}{4} = \frac{2}{1}$.

PAGE 235

14. Draw QA and $SB \perp PT$.
Prove $\triangle QAR = \triangle PBS$ (§ 110). $\therefore QA = SB$.

15. 80; 8000 sq. ft.

1. 16 : 25.

PAGE 238

1. 13 in. **2.** 15 ft.

3. Other side of rectangle $= \sqrt{\overline{20}^2 - \overline{16}^2} = 12$. $\therefore$ 192 sq. in. *Ans.*

4. Let x = side of square. $\therefore x^2 + x^2 = 100$. $\therefore x^2 = 50$. *Ans.*

5. Alt. $= \sqrt{\overline{24}^2 - \overline{12}^2}$ in. (§ 356) $= 12\sqrt{3}$ in. $= 20.78 +$ in. *Ans.*

PAGE 239

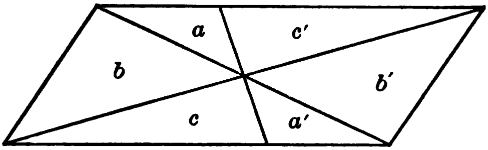
6. By the method of Ex. 5, p. 238, find altitude of ▱ $= 5\sqrt{3}$. $\therefore$ Area $= 100\sqrt{3}$ sq. ft. $= 173.2 +$ sq. ft. *Ans.*

7. Let 24 in. be the base. Denote the altitude by x. Then $x^2 + x^2 = 18^2$. $\therefore x = 9\sqrt{2}$. $\therefore$ Area $= 108\sqrt{2}$ sq. in. or $152.73 +$ sq. in. *Ans.*

8. $a\sqrt{b^2 - a^2}$.

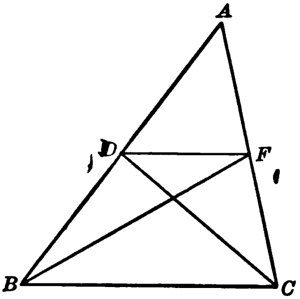
9. Distance $= \sqrt{\overline{60}^2 + \overline{18}^2}$ ft. $= 62.64$ ft. *Ans.*

1. (Group 63).



Show that $\triangle a = \triangle a'$ (§ 80), $\triangle b = \triangle b'$ (§ 79), $\triangle c = \triangle c'$ (§ 80). Add, etc.

2.



$DF \parallel BC$ (§ 300) $\therefore \triangle ABC \sim \triangle ADF$ (§ 305) $\therefore$ by § 352 $\frac{\triangle ABC}{\triangle ADF} = \frac{\overline{AC}^2}{\overline{AF}^2} = \frac{(2\overline{AF})^2}{\overline{AF}^2} = \frac{4}{1}$. Or, in $\triangle ABC$, let $AB = 2AD$, $AC = 2AF$. $\frac{\triangle ABC}{\triangle ADC} = \frac{2}{1}$ (§ 345). $\frac{\triangle ADC}{\triangle ADF} = \frac{2}{1}$ (§ 345). $\therefore \frac{\triangle ABC}{\triangle ADF} = \frac{4}{1}$ (Ax. 4).

3. See Ex. 2, and § 156.

4. See figure of Ex. 2. Show that the $\triangle$ ADF, DFB, and DFC are all equal in area.

5. Use figure, p. 206 (text-book). Area of $\triangle ABC = \frac{1}{2}AB \times BC$ (§ 343) or $= \frac{1}{2}AC \times BF$ (§ 343). $\therefore \frac{1}{2}AB \times BC = \frac{1}{2}AC \times BF$ (Ax. 1). $\therefore AB \times BC = AC \times BF$ (Ax. 4).

6. $\frac{1}{2}b \times a = \frac{1}{2}b' \times 3a$ (Hyp.). $\therefore b' : b = a : 3a = 1 : 3$ (§ 281).

7. Denote one of the legs by c and half the base of the first $\triangle$ (and $\therefore$ the altitude of the second $\triangle$) by b. Then the area of each $\triangle = b\sqrt{c^2 - b^2}$ (§§ 356, 343).

9. Two trapezoids are formed whose corresponding bases and whose altitudes are equal. Thus,
$\frac{1}{2}(\frac{1}{2}b + \frac{1}{2}b')h = \frac{1}{2}(\frac{1}{2}b + \frac{1}{2}b')h$ (§ 350).

PAGE 240

10. Area of $\triangle AOB$ = area of $\triangle AOD$ (§ 344). Area of $\triangle BOC$ = area of $\triangle DOC$ (§ 344). $\therefore$ area of $\triangle ABC$ = area of $\triangle DAC$ (Ax. 2).
If one diagonal of a parallelogram bisects the other diagonal, the first diagonal divides the parallelogram into two equivalent triangles.

11. $\triangle ABC = \triangle ADC$ (1), $\triangle PTC = \triangle PRC$ (2), $\triangle AQP = \triangle ASP$ (3) (§ 156). Add (3) and (2) and subtract the result from (1) (Axs. 2, 3).

12. Let $ABCD$ be the given quadrilateral, and P the midpoint of the diagonal AC. Then $\triangle ABP = \triangle PBC$ (§ 344). Also $\triangle ADP = \triangle PDC$ (§ 344). Adding, quad. $ABPD$ = quad. $PBCD$ (Ax. 2).

1. (Group 64). See figure of Ex. 1 (Group 51), p. 210 (text-book). $\overline{AB}^2 = \overline{AD}^2 + \overline{DB}^2$ (§ 355). Also $\overline{AC}^2 = \overline{AD}^2 + \overline{CD}^2$ (§ 355). Subtract.

2. Let $ABCD$ be the given quadrilateral and let the diagonals AC and BD intersect at right $\angle$s at the point O. Then, by § 355, $\overline{AB}^2 = \overline{BO}^2 + \overline{OA}^2$, $\overline{CD}^2 = \overline{OC}^2 + \overline{OD}^2$. $\therefore \overline{AB}^2 + \overline{CD}^2 = \overline{BO}^2 + \overline{OA}^2 + \overline{OC}^2 + \overline{OD}^2$. Also $\overline{BC}^2 = \overline{BO}^2 + \overline{OC}^2$, $\overline{AD}^2 = \overline{AO}^2 + \overline{OD}^2$. $\therefore \overline{BC}^2 + \overline{AD}^2 = \overline{BO}^2 + \overline{OA}^2 + \overline{OC}^2 + \overline{OD}^2$. Use Ax. 1.

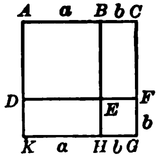
3. The altitude bisects the base (§ 117). Denote one side of the $\triangle$ by a, and the altitude by x. $\therefore a^2 = x^2 + \left(\frac{a}{2}\right)^2$ (§ 355). $\therefore x^2 = \frac{3a^2}{4}$.

4. AB is the hypotenuse. $\therefore \overline{AB}^2 = \overline{AC}^2 + (2\overline{CK})^2 = \overline{AC}^2 + 4\overline{CK}^2$. Also $\overline{AK}^2 = \overline{AC}^2 + \overline{CK}^2$ (§ 355). Subtract, etc.

5. $\overline{AQ}^2 = \overline{AC}^2 + \overline{CQ}^2$, $\overline{BP}^2 = \overline{PC}^2 + \overline{BC}^2$ (§ 355). Adding, $\overline{AQ}^2 + \overline{BP}^2 = \overline{AC}^2 + \overline{BC}^2 + \overline{PC}^2 + \overline{CQ}^2$ (Ax. 2) $= \overline{AB}^2 + \overline{PQ}^2$ (Ax. 9).

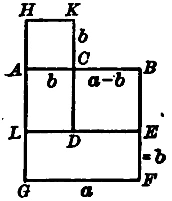
6. $\overline{BE}^2 = \overline{AB}^2 + \overline{AE}^2 = \overline{AB}^2 + \frac{1}{4}\overline{AC}^2$ (§ 355). $\overline{CF}^2 = \overline{AF}^2 + \overline{AC}^2 = \frac{1}{4}\overline{AB}^2 + \overline{AC}^2$ (§ 355). $\therefore \overline{BE}^2 + \overline{CF}^2 = \frac{5\overline{AB}^2}{4} + \frac{5\overline{AC}^2}{4}$ (Ax. 3). $4(\overline{BE}^2 + \overline{CF}^2) = 5(\overline{AB}^2 + \overline{AC}^2) = 5BC^2$ (Ax. 9).

7.



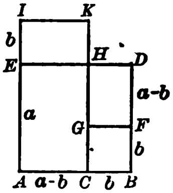
Let line AB be denoted by a, BC by b; let AG be the square on AC, and AE the square on AB. Then $EG = b^2$, and rectangles BF and EK each $= ab$. $\therefore BF = EK$. But $AG = AE + EG + 2BF$, or $(a + b)^2 = a^2 + b^2 + 2ab$.

8.



Let $AB = a$, $AC = b$. Then $BC = a - b$. Let AF be the square on AB, CE the square on CB, and AK the square on AC. Hence, the rectangles LK and GE each $= ab$. Then $BD = AF + AK - 2GE$, or $(a - b)^2 = a^2 + b^2 - 2ab$.

9.



Let $AB = a$, $BC = b = IE$; then $DF = a - b$. Let AD be the square on AB, and CF the square on CB. Then the rectangle $EK = DG$ (for each $= b(a - b)$). But rectangle $AK = AD + EK - DG - CF = AD - CF$. $\therefore (a + b)(a - b) = a^2 - b^2$.

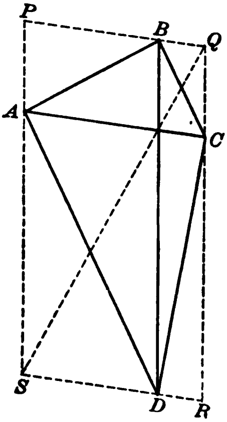
10. 1 : 6.

11. 31,875 sq. mi.

PAGE 241

1. Draw the altitude of the ▱ through P and $\perp AD$, and denote it by h. Then by § 343 and Ax. 2, $\triangle PAD + \triangle PBC = \frac{1}{2}AD \times h = \frac{1}{2}$ ▱ $ABCD$ (§ 339), etc.

2. Draw a $\perp$ from P to AD. Use § 355 (four times), Ax. 2, etc.
3. See method of proof in § 386.
4. Show that the top and bottom $\triangle$ together $= \frac{1}{2}$ the trapezoid, *i.e.* $= \frac{1}{2}h(b_1 + b_2)$.
5. See Ex. 4.
6. $\overset{\frown}{AC} + \overset{\frown}{BD} =$ semicircle (for $\frac{1}{2}$ their sum measures the rt. $\angle O$). Also $\overset{\frown}{BD} + \overset{\frown}{DE} =$ semicircle (constr.). $\therefore \overset{\frown}{AC} + \overset{\frown}{BD} = \overset{\frown}{BD} + \overset{\frown}{DE}$ (Ax. 1). $\therefore \overset{\frown}{AC} = \overset{\frown}{DE}$ (Ax. 3). $\therefore$ chord $AC =$ chord DE (§ 200). $\therefore \overline{OA}^2 + \overline{OC}^2 = (\text{chord } AC)^2 = (\text{chord } ED)^2$ (§ 355). But $\overline{OB}^2 + \overline{OD}^2 = (\text{chord } BD)^2$ (§ 355). $\therefore \overline{OA}^2 + \overline{OC}^2 + \overline{OB}^2 + \overline{OD}^2 = (\text{chord } ED)^2 + (\text{chord } DB)^2 = \overline{EB}^2$.
7. Denote the vertices of the quadrilateral by A, B, C, D, and the midpoints of AB, BC, CD, DA by 1, 2, 3, 4, respectively. Let the diagonals BD and AC intersect in O. Draw $O1$. Let 12 intersect BD in P. Then $OP = PB$ (§ 296). $\therefore \triangle O1P = \triangle 1PB$ (§ 344). Let 14 intersect AO in Q. Then similarly $\triangle O1Q = \triangle A1Q$, etc.
8.



Let $ABCD$ be the given quadr. Through B draw $PQ \parallel$ the diagonal AC, through D draw $SR \parallel AC$, through C draw $QR \parallel BD$, through A draw $PS \parallel BD$. Then show $ABCD = \frac{1}{2}PQRS$. Also $\triangle SPQ = \frac{1}{2}PQRS$ (§ 156). $\therefore ABCD = \triangle PQS$. But $PQ = AC$ (§ 157), etc.

PAGE 242

1. The base of ▱ = median of the trapezoid (§ 157). Use § 339, Ex. 7, p. 213.
2. Join the vertex with the point in base. Use two pairs of △s equal in area. Add.
3. Draw ⊥s from the other vertices of the ▱, viz.: *B* and *D*, to the diagonal *AC*. See Ex. 11, p. 93.
4. The altitude $= \sqrt{a^2 - \frac{a^2}{4}} = \frac{a\sqrt{3}}{2}$ (§ 356), etc.
5. By Ex. 4, if the side of the second △ is a, the side of the first △ is $\frac{a\sqrt{3}}{2}$. $\therefore\ K : K' = \left(\frac{a\sqrt{3}}{2}\right)^2 : a^2$ (§ 352) $= \frac{3a^2}{4} : a^2 = 3 : 4$.
6. Denote side of △ by a and draw lines from the given point to the vertices of the △. Then area of △ $= \frac{1}{2}a \times$ (sum of ⊥s) (§ 343, Ax. 2). But area of △ $= \frac{1}{2}a \times h$ (§ 343). $\therefore\ \frac{1}{2}a \times$ (sum of ⊥s) $= \frac{1}{2}a \times h$ (Ax. 1). ∴ sum of ⊥s $= h$ (Ax. 5).
7. See Ex. 10, p. 240.
8. In the △ in which the included angle is obtuse, produce the base through the vertex of the obtuse angle, till the produced part = base, etc.
9. Use § 345, Ax. 4.
10. △ *QDC* = ½ ▱ *ABCD* (same base *CD*, etc.). △ *ABP* + △ *PCD* = ½ ▱ *ABCD*, etc. Use Ax. 3.

PAGE 243

1. Construct a right triangle whose legs are the sides of the given squares.
2. Reduces to Ex. 12, p. 139.
3. See Ex. 1.
4. See Ex. 1.
5. See Ex. 1.
6. Draw a line through the vertex ∥ the base. Bisect base; at the midpoint of the base erect a ⊥ to the base, etc.
7. Draw a line through the vertex ∥ to the base. At one end of the base construct an ∠ = given ∠, etc.
8. Draw a line through the vertex ∥ the base. Using one end of the base as a center and the required side as a radius, describe an arc, etc.
9. See Ex. 7.

10. Let b be the base of the given $\triangle$, and x that of the required $\triangle$. $\therefore\ b^2 : x^2 = 1 : 2$. $\therefore\ x = b\sqrt{2}$ (see Ex. 2, p. 208).

11. Let ABC be the given triangle. Construct any square and draw its diagonal. Then divide the side BC into segments at G such that $BC : BG =$ diagonal of square : side of square. Through G draw a line $\parallel AC$, etc.

12. Draw the $\perp$ through the intersection of the diagonals. (See Ex. 1, p. 239.)

13. Draw the line through the given point and through the intersection of the diagonals. (See Ex. 1, p. 239.)

14. Use § 321.

PAGE 244

1. $\frac{2 \times 36}{4 \times 6} = \frac{3}{1}$. *Ans.* $\frac{b''}{a''}$. *Ans.* 2. 4.

3. Since 6 = radius of the log, a side of the square beam $= \sqrt{6^2 + 6^2} = 6\sqrt{2}$. On the diagram of Ex. 6, p. 219, $AB = 12$, $DB = 4$. $\therefore\ FB = \sqrt{4 \times 12} = 4\sqrt{3}$ (§ 320). $FB : AF = 1 : \sqrt{2}$, or $4\sqrt{3} : AF = 1 : \sqrt{2}$. $\therefore\ AF = 4\sqrt{6}$. $\frac{(4\sqrt{6})^2 \times 4\sqrt{3}}{(6\sqrt{2})^2 \times 6\sqrt{2}} = \frac{8\sqrt{3}}{9\sqrt{2}} = \frac{13.85+}{12.72+}$. *Ans.*

4. When the width of the beam is 3 in., by § 320 we find height of beam $= 3\sqrt{15}$. $\therefore$ Required ratio $= \frac{(4\sqrt{6})^2 \times 4\sqrt{3}}{(3\sqrt{15})^2 \times 3} = \frac{128\sqrt{3}}{135} = \frac{221.69+}{135}$. *Ans.*

5. Denote the offsets by $a_1, a_2, a_3, \ldots a_n$, and the common distance between them by h. Then the sum of the areas of the small trapezoids forming the figure $= \frac{1}{2}h(a_1 + a_2) + \frac{1}{2}h(a_2 + a_3) + \ldots + \frac{1}{2}h(a_{n-1} + a_n) = \frac{1}{2}h(a_1 + a_n) + h(a_2 + a_3 + \ldots + a_{n-1})$.

6. In Ex. 5, if the initial and final offsets reduce to zero the formula becomes $h(a_2 + a_3 + \ldots + a_{n-1}1)$. In the diagram of Ex. 6 draw the longest possible chord to the irregular curve and divide this chord into a suitable number of equal parts. At the points of division erect $\perp$s to meet the curve. Then the entire area approximately equals the sum of the $\perp$s multiplied by the common distance between them.

7. Let $FG = 100$ ft., $GK = 140$ ft., $KL = 80$ ft., $FB = 160$ ft., $GC = 220$ ft., $KD = 220$ ft., $LE = 140$ ft., $HA = HG = 50$ ft. Then area of $ABCD = \frac{100}{2}(160 + 220) + 140 \times 220 + \frac{80}{2}(220 + 140) - \frac{150}{2}(160 + 50) - \frac{170}{2}(50 + 140) = 32{,}300$ sq. ft. *Ans.*

8. Let O be the center of $\overset{\frown}{BC}$. At B erect $BO \perp AT$. Draw Ob_3. From b_3 draw $b_3H \perp OB$. Then $OH = \sqrt{r^2 - (3d)^2}$. $\therefore a_3b_3 = BH = r - \sqrt{r^2 - (3d)^2}$. In like manner, $a_nb_n = r - \sqrt{r^2 - (nd)^2}$.

PAGE 245

9. Speed per hour $= \sqrt{12^2 + 3^2}$ mi. $= 12.36 +$ mi.

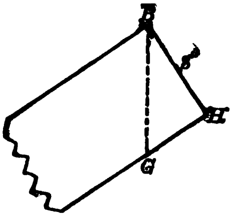
12. 500 lb.

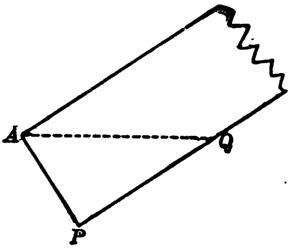
14. 5.66 + mi. per hour.

15. 25 mi. per second.

16. Use the tape to make a right triangle whose sides are 24 ft., 32 ft., 40 ft.

17.





$AD = 10$ ft., $BD = 6\frac{2}{3}$ ft., $\therefore$ AB $= 12.01 +$ ft.
At the peak, GH, the part cut off from the lower edge of the rafter, is $\frac{2}{3}$ of BH. To determine the line along which, at the lower end of the rafter, the lower edge of the rafter must be cut, $PQ = \frac{2}{3}\,AP$, where AP is the width of the rafter.

PAGE 246

3. §§ 343, 344, 345, 346, 352, 355, 356.

4. §§ 334, 335, 336, 337.

5. §§ 339, 340, 341, 342, 347.

6. § 348.

7. § 350.

8. §§ 353, 354.

9. See § 333.

10. "The number of units in the area of a rectangle divided by the number of linear units in the base."

11. When two sides of a right triangle are known, Prop. X enables us to find the third side without the labor of measuring it.

14. See p. 280.

15. Correct for square and rectangle. It gives too large a result for all other quadrilaterals.

PAGE 248

1. 9 : 64.

PAGE 251

1. 108°. **2.** Use § 79.

PAGE 252

7. Side of square $= \sqrt{3^2 + 3^2}$ in. $= 3\sqrt{2}$ in. $= 4.23 +$ in. Area $= 18$ sq. in. *Ans.*

8. $r^2 + r^2 = (\frac{3}{2})^2$. $\therefore r = 1.06 +$ in. *Ans.*

9. $7\sqrt{2}$ in. $= 9.89 +$ in. *Ans.*

10. Use § 121. **11.** Use § 345. **12.** § 363.

PAGE 253

1. 72°. **2.** 60°. **3.** 90°.

4. $5\sqrt{2}$ in. $= 7.07 +$ in. *Ans.* Area $= 200$ sq. in. *Ans.*

5. Apothem $= 5\sqrt{3}$ in. $= 8.66 +$ in. *Ans.* Area $\frac{3\sqrt{3}}{2} \times 100$ sq. in. $= 259.8 +$ sq. in. *Ans.*

6. $\frac{1}{2}b\sqrt{3}$; $\frac{3\sqrt{3}b^2}{2}$.

7. $\frac{1}{2}b\sqrt{2}$; $2b^2$.

8. 4 in.

9. $\overset{\frown}{FA} = \overset{\frown}{FB}$ (§ 202) $\therefore FA = FB$ (§ 363), etc.

10. If apothem $= x$, $r = 2x$ (Ex. 9). $\therefore (2x)^2 - x^2 = 9$. $\therefore x = \sqrt{3}$ in. Area $= 9\sqrt{3}$ sq. in. *Ans.*

11. See Ex. 10. $r = 4$. *Ans.*

12. Circumscribe a circle about the given polygon. Use §§ 198, 235.

13. Area $= 584.56 +$ sq. ft.

PAGE 255

5. 400 sq. in.

6. 129.90 sq. in.; 519.61 sq. in.

7. Denote the radius of the circle by r. Then area of hexagon $= \frac{3\sqrt{3}r^2}{2} = 2\left(\frac{3\sqrt{3}r^2}{4}\right) = 2$ area of triangle.

8. $\frac{2r^2}{4r^2} = \frac{1}{2}$. *Ans.*

9. 3 : 4.

10. On the diagram of p. 249, draw straight lines connecting the alternate vertices.

11. In each of at least two adjacent squares on the squared paper, inscribe a regular octagon (see Ex. 11, p. 270). The figure is completed by drawing lines parallel to the sides of these octagons, which lines will form sides of inscribed octagons in the other squares composing the squared paper.

PAGE 256

1. $128\sqrt{3}$ sq. in. = 221.70 + sq. in.
2. Use §§ 241, 102, etc.
3. Use Ex. 2 and § 352.
4. See Ex. 9, p. 253.

PAGE 257

1. 1 : 2; 1 : 4.
2. 4 : 9; 2 : 3; 2 : 3.
3. Use §§ 376, 353. ∴ 414.72 sq. in. *Ans.*
4. 16 : 25; 16 : 25.

PAGE 258

1. (§ 379). $10\sqrt{3}$.

3. 8.04 − in. *Ans.* The perimeters and their difference would each be doubled. 16.07 + in. *Ans.*

PAGE 259

1. 4 : 1.
2. $27\sqrt{3}$ sq. in. or 46.764 + sq. in.; 25.236 + sq. in.

PAGE 261

1. 88 in.
2. 66 in.
3. 11 ft.
4. $7\frac{3}{7}$ yd.
5. $10\frac{3}{4}$ ft.
6. .0993 + in.
7. Two places.
8. $r = 14$ in.
9. $r = 2.1$ in.
10. $r = \frac{1}{6}$ in.
11. $r = 9$.
12. 2 : 3 (§ 381).
13. 5 : 7.
14. $7\frac{1}{3}$ in.
15. $9\frac{1}{6}$ in.

16. $\frac{117}{360} \times 44$ in. = 14.3 in. *Ans.*

17. $2\frac{281}{360}$ in.

18. 8.

19. Circf. = 132 in. $\frac{33}{132}$ of 360° = 90°. *Ans.*

20. $22\frac{1}{2}°$.

21. $152\frac{8}{11}°$.

22. 3.

PAGE 262

1. Make the arc greater than a semicircle.

PAGE 264

1. 616 sq. in.

2. 346.5 sq. in.

3. .000616 sq. ft.

4. .0616 sq. in.

5. $5\frac{211}{224}$ sq. in.

6. $7\frac{1}{14}$ sq. in.

7. $\frac{22a^2}{7}$ sq. ft.

8. $\frac{11a^2}{224}$ sq. ft.

9. 2464 sq. ft.

10. .1386 sq. in.

11. $\frac{504}{1331}$ sq. in.

12. $1018\frac{2}{7}$.

13. 40.26 sq. in.

14. 100 : 1; 10 : 1; 10 : 1.

15. 4 times (§ 393).

16. $56\frac{4}{7}$ in.

17. 21.

18. $4b$.

19. 20.

20. 5.64 + sq. ft.

PAGE 265

21. $102\frac{2}{3}$ sq. in.; $136\frac{8}{9}$ sq. in.; $35\frac{14}{15}$ sq. in.

22. πb^2; $\frac{\pi b^2}{4}$; $4\pi b^2$.

23. $\frac{\pi R^2}{4}$; $4\pi R^2$; $3\pi R^2$.

24. $\frac{\pi R^2}{16}$; $\frac{\pi R^2}{2}$; $\frac{\pi R^2}{8}$.

25. 9.00 + in.

26. Measure the circumference of the pipe and divide the result by π.

27. Circf. of the first wheel = 6 times the circf. of the second. $\therefore$ 6 × 120 = 720. *Ans.* 300 ÷ 6 = 50. *Ans.*

28. It becomes $\frac{4}{9}$ of what it was (since $(1\frac{1}{2})^2 : 1^2 = \frac{9}{4} : 1$).

29. $3^2 : x^2 = 1 : 2$. $\therefore$ $x = 3\sqrt{2} = 4.24 +$. $\therefore$ $4\frac{1}{4}$ in. *Ans.*

30. $\overline{12}^2 : \overline{18}^2 = 2^2 : 3^2 = 4 : 9$. $\therefore$ $40{,}000 \times \frac{9}{4} = 90{,}000$. *Ans.*

31. (1) $\frac{9}{16}$ of 50,000 lb. = 28,125 lb. *Ans.*
(2) $1 : x^2 = 50{,}000 : 150{,}000$. $\therefore$ $x^2 = 3$. $\therefore$ 1.732 + in. *Ans.*

PAGE 266

1. A line $\perp$ to both the given lines (§§ 137, 100).

2. Diagonal $= \sqrt{12^2 + 5^2}$ ft. $= 13$ ft. long. *Ans.*

PAGE 267

3. Side of square $= \frac{1}{8}$ mi. $\therefore$ Area $= \frac{1}{64}$ sq. mi. $= 10$ acres. Radius of circle $= \frac{7}{88}$ mi. $\therefore$ area of circle $= \frac{7}{352}$ sq. mi. $= 12\frac{8}{11}$ acres. $\therefore$ $2\frac{8}{11}$ acres. *Ans.*

4. 36 sq. in. In the second $\triangle$, alt. $= 3\sqrt{3}$. $\therefore K = 18\sqrt{3}$ sq. in. $= 31.176+$ sq. in.

5. 200 ft., 2400 sq. ft.; 200 ft., 2500 sq. ft.

6. 6.928 + sq. in.; 9 sq. in.; 10.392+ sq. in.; 11.45 + sq. in. For, by Ex. 4, p. 242, $a = 4$. $\therefore K$ of $\triangle = \dfrac{16\sqrt{3}}{4} = 6.928+$. By § 362, K of a regular hexagon, whose side is a, $= \dfrac{6a^2\sqrt{3}}{4} = \dfrac{3a^2\sqrt{3}}{2}$. $\therefore K$ of given hexagon $= \dfrac{3 \times 2^2\sqrt{3}}{2} = 6\sqrt{3} = 10.392+$. For method of finding area of $\odot$ see Ex. 9, p. 264.

7. 22.33+ in.; 19.593+ in.; 17.05+ in. By Ex. 4, p. 242, in given $\triangle$, $\dfrac{a^2\sqrt{3}}{4} = 24$. $\therefore a = 7.43+$. Side of square $= \sqrt{24}$ $= 4.899+$. For circle, $\pi R^2 = 24$. $\therefore R = \sqrt{\dfrac{84}{11}}$, etc.

1. (Group 76). One; three.

2. Four; five.

3. Six.

4. Seven; eight; n; an infinite number.

5. *A B D E H I M O T U V W X Y.*

PAGE 268

1. No; yes.

2. No; yes.

3. Regular polygons of an even number of sides have a center of symmetry; regular polygons of an odd number of sides do not.

4. Yes.

6. Two (the diagonals); yes.

7. One diagonal; no, except when the figure is a rhombus.

8. Draw any line through this point and terminated by the perimeter. Show that it is bisected (§ 80).

9. Diameter ⊥ the chord of the segment.

10. No; no.

11. Two; yes.

12. Line of centers.

13. *I N O X Z*; *o s x z*.

1. (Group 78). §§ 367, 235, 304.

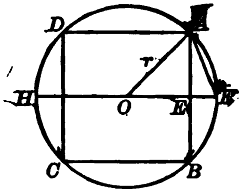
2. Circumscribe a circle about the given pentagon (§ 367). $\angle KBC \overset{m}{=} \frac{1}{2}\widehat{EDC}$, and $\angle BKC \overset{m}{=} \frac{1}{2}(\widehat{AE} + \widehat{BC})$. $\therefore \angle KBC = \angle BKC$, etc.

3. Find side of △ is $r\sqrt{3}$ (see Ex. 9, p. 253), etc.

4. $2r^2 - \dfrac{3\sqrt{3}r^2}{4}$ or $r^2(2 - \frac{3}{4}\sqrt{3})$.

PAGE 269

5.



In the circle *O*, let *ABCD* be an inscribed square, and *F* the mid-point of the arc *AB*. ∴ chord *AF* = side of inscribed regular octagon (§ 361). $OE = \sqrt{r^2 - \overline{AE}^2} = \sqrt{r^2 - \frac{r^2}{2}} = \frac{r}{2}\sqrt{2}$.

$\therefore EF = r - \frac{r}{2}\sqrt{2}$.

Then, by § 320, $AF = \sqrt{HF \times EF} = \sqrt{2r\left(r - \frac{r}{2}\sqrt{2}\right)}$

$$= r\sqrt{2\left(1 - \frac{\sqrt{2}}{2}\right)} = r\sqrt{2 - \sqrt{2}}.$$ *Ans.*

6. Follow the method used in Ex. 5.

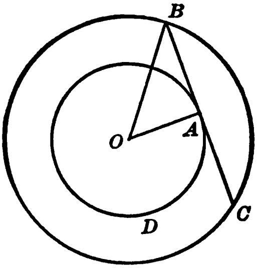
7. See Ex. 11, p. 252. Side $= \sqrt{\frac{a^2}{4} + \frac{a^2}{4}} = \frac{1}{2}a\sqrt{2}$. *Ans.*

8. 1 : 3.

9. The squares formed include between them six equilateral △ each of whose sides is *b*. ∴ area of dodecagon $= (3\sqrt{3} + 6)b^2$.

10. § 393.

11.



Let OAD and OBC be the two circles forming the circular ring. Denote the radius of the larger circle by R and that of the smaller circle by r. $\therefore$ area of ring $= \pi(R^2 - r^2)$. But in the right $\triangle$ OAB, $\overline{OB}^2 - \overline{OA}^2 = \overline{AB}^2$, *i.e.*, $R^2 - r^2 = \overline{AB}^2$. $\therefore$ area of ring $= \pi\overline{AB}^2$, etc.

12. Denote AB by q, BC by p, AC by r. The sum of the crescents $= \triangle\, ABC + AcB + BdC - AaBbC = \frac{1}{2}pq + \frac{\pi q^2}{8} + \frac{\pi p^2}{8} - \frac{\pi r^2}{8}$ (§ 390) $= \frac{1}{2}pq + \frac{\pi}{8}(q^2 + p^2 - r^2)$. But $r^2 = p^2 + q^2$ (§ 355). $\therefore$ sum of crescents $= \frac{1}{2}pq = \triangle\, ABC$.

13. $\therefore$ in like manner side BC = side DE = side AB = side DC = side AE. (Use §§ 235, 357, Ax. 3, § 200.)

14. Prove the right $\triangle$s thus formed equal by § 216, Ax. 5, § 111.

15. Denote the center by O. Then $PT = PQ$ (Hyp.), $PA = PB$ (§ 216). $\therefore$ $AT = BQ$ (Ax. 3). $\therefore$ rt. $\triangle\, OAT$ = rt. $\triangle\, OBQ$ (§ 79). $\therefore$ $\angle T = \angle Q$ (§ 216, Ax. 4). Similarly, $\angle Q = \angle S = \angle P = \angle R = \angle T$.

PAGE 270

1. Denote the radii of the given circles by R and r. Then the new radius $= R + r$.

2. The new radius $= R - r$.

3. The new radius $= \sqrt{R^2 + r^2}$. See Ex. 1, p. 243.

4. New radius $= \sqrt{R^2 - r^2}$. See Ex. 2, p. 243.

5. The trees must be arranged at the vertices of a regular octagon of which each side is 20 ft. By Ex. 5, p. 269, $20 = r\sqrt{2 - \sqrt{2}}$,

whence $r = 10\sqrt{2 + 2\sqrt{2}}$. Hence, if a circle is described with a radius of one inch, and a regular octagon is inscribed in the circle, the drawing will answer the given conditions and have a scale of 1 in. to $10\sqrt{2 + 2\sqrt{2}}$ ft.

6. Let x be the radius of the new circle. Then $\pi x^2 : \pi r^2 = 1 : 2$. $\therefore\ x = r\dfrac{\sqrt{2}}{2}$, etc.

7. Produce the radii of the given sector, and draw a tangent at the midpoint of the arc of the sector. Inscribe a ⊙ in the △ thus formed (§ 259).

8. Construct any square, and join the midpoint of one side of the square with a vertex which is not one of the extremities of the side taken. At the midpoint of the chord of the given segment construct an $\angle = \angle$ formed between the line drawn in the square and the side of the square whose midpoint is taken, etc.

9. Draw the bisectors of the three ⦞ of the given △. In each △ thus formed inscribe a ⊙ (§ 259).

10. Divide the given ⊙ into three equal sectors (§ 362), and inscribe a circle in each sector (Ex. 7).

11. Draw lines from the center of the given square ⊥ sides, and also to the vertices. Bisect the ⦞ thus formed, etc.

12. Denote the radius of the given semicircle by R, and that of the required circle by x. Then $\pi x^2 = \frac{1}{2}\pi R^2$. $\therefore\ x^2 : R^2 = 1 : 2$. $\therefore\ x = \dfrac{\sqrt{2}R}{2}$.

1. (Group 80). $2\pi x + 2y = 2640$, $2x + y = 1000$. Hence, $x =$ 280 ft., $2x = 560$ ft. *Ans.*

PAGE 271

2. Use § 385. $\dfrac{5\frac{1}{6}^\circ}{180^\circ}\left(\dfrac{22r}{7}\right) = 10$. $\therefore\ r = 110.8 +$ ft. *Ans.*

3. Limit of speed $= 5000 \times 12$ in. per minute. Circf. of wheel $= \frac{22}{7} \times 27$ in. $\therefore$ no. rev. per min. $= 60{,}000$ in. $\div (\frac{22}{7} \times 27$ in.$)$ $= 707 +$. *Ans.*

5. Point of intersection of the diagonals of the square; point of intersection of the diagonals; center of the polygon; center.

6. Center of the square.

7. Ratio $= \frac{\pi(3^2)3}{\pi(6^2)6} = \frac{27}{216} = \frac{1}{8}$. *Ans.*

8. The area of the circular ring which forms the cross-section of the hollow tube $= \pi(R^2 - r^2)$. Hence, the radius of a solid cylindrical beam having an equivalent cross-section would be $\sqrt{R^2 - r^2}$. Use law of strains stated in Ex. 7. Hence, ratio =
$$\frac{\pi(R^2 - r^2)\left(\frac{R^2 + r^2}{R}\right)}{\pi(R^2 - r^2)\sqrt{R^2 - r^2}} = \frac{\pi 7\left(\frac{16+9}{4}\right)}{\pi 7(\sqrt{7})} = \frac{25}{4\sqrt{7}} = 2.36 +. \quad \textit{Ans.}$$

PAGE 272

11. If $OA = r$, $OC = \frac{5r}{4}$. $\therefore$ area of square $= \frac{25r^2}{8}$, while the area of the circle is $\frac{22r^2}{7}$. Hence, the approximation described in Ex. 11 consists essentially in using $\frac{25}{8}$ as the value of π. To obtain the per cent of error, we have $\left(\frac{22}{7} - \frac{25}{8}\right) \div \frac{22}{7} = \frac{1}{56} \times \frac{7}{22}$ $= .0056 +$. Hence, a little more than $\frac{1}{2}$ per cent. *Ans.*

PAGE 273

1. See § 364.

2. Radius; diameter; circumference.

3. It reduces the measurement of the area of a circle to the measurement of a single straight line.

4. After we have computed the ratio of the circumference to the diameter in any one circle, by the aid of Prop. IX we know the value of this ratio in every other circle without the labor of any further computation.
Prop. X reduces the measurement of the area of a regular polygon to the measurement of two straight lines, viz.: one side and the apothem.

PAGE 274

5. Circles aid (1) in the construction of regular polygons. See §§ 360, 361, 362, 364, 373, etc. (2) In determining whether a given polygon is regular. See § 359. (3) In proving the properties of regular polygons. See Ex. 2 (Group 78), p. 268.

6. (1) In proving properties of a single circle and, hence, of parts of a circle. §§ 379, 390, 394. (2) In proving properties of two or more circles. §§ 380, 382, 393.

7. § 376.

8. $d : d' = c : c'$; $r : r' = c : c'$; $K : K' = c^2 : c'^2$. (Use § 291.)

9. It is equal to it; ½.

11. The polygons are similar (§ 376). $\therefore$ $P : P' = a : a'$ (§ 317); also $r : r' = a : a'$ (§ 318). Ratio of apothems $= a : a'$ (§ 318). $K : K = a^2 : a'^2$ (§ 353).

12. §§ 357, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 372.

13. §§ 376, 377, 378.

14. §§ 382, 383, 384, 390, 391, 392, 397.

15. §§ 379, 380, 381, 393.

PAGE 284

1. $x = \sqrt{\overline{50}^2 - \overline{40}^2}$ ft. $= 30$ ft. *Ans.*

2. $\sqrt{\overline{12}^2 + 5^2} = 13$. *Ans.*

3. The altitude of an isosceles △ bisects the base (§ 110). Hence, altitude $= \sqrt{5^2 - 4^2} = 3$. *Ans.*

4. See § 121. $\sqrt{\overline{17}^2 - \overline{15}^2} = 8$. Hence, the other diagonal $= 16$. *Ans.*

5. Longest chord = the diameter, or 10. The diameter through the given point is ⊥ shortest chord (§§ 208, 136). $\therefore$ half of the shortest chord (§ 202) $= \sqrt{5^2 - 3^2} = 4$. $\therefore$ shortest chord $= 8$. *Ans.*

6. Use figure p. 118 (text-book). Let $OA = 25$ in., $AB = 48$ in. Then $AR = 24$ in. (§ 202). $\therefore$ $OR = \sqrt{\overline{25}^2 - \overline{24}^2}$ in. $= 7$ in. *Ans.*

7. Use figure p. 122 (text-book). Draw OD and OB. Let $CD =$ 12 in., $OG = 5$ in., $AB = 10$ in. Then $GD = 6$ (§ 202). $\therefore$ $OD =$ $\sqrt{6^2 + 5^2}$ in. $= \sqrt{61}$ in. Also $FB = 5$ in. (§ 202). $\therefore$ $OF =$ $\sqrt{61 - 25}$ in. $= 6$ in. *Ans.*

8. $\sqrt{\overline{40}^2 - \overline{20}^2} + \sqrt{\overline{40}^2 - \overline{30}^2} = 61.09+$. $\therefore$ $61.09+$ ft. *Ans.*

9. Denote the hypotenuse by $2x$. Then the unknown leg $= x$. $\therefore$ $4x^2 - x^2 = 100$. $\therefore$ $2x = \frac{20\sqrt{3}}{3} = 11.547+$. *Ans.*

10. Alt. $= \sqrt{36 - 9} = \sqrt{27} = 5.196 +$. *Ans.*

11. The altitude bisects the base (§ 117). Denote a side of the given △ by $2x$. Then $4x^2 - x^2 = 8^2$ (§ 355). $\therefore 2x = \frac{16\sqrt{3}}{3} = 9.237 +$. *Ans.*

12. Denote a side of the square by x. Then $x^2 + x^2 = \overline{15}^2$. $\therefore x = \frac{15\sqrt{2}}{2} = 10.606 +$. *Ans.*

13. Denote the hypotenuse by x. Then $9 - x =$ the other leg. $\therefore x^2 - (9 - x)^2 = 9$. $\therefore x = 5, 9 - x = 4$. *Ans.*

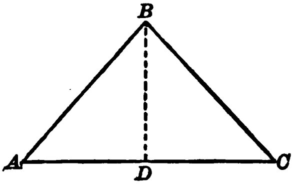
14. See Ex. 21, p. 174. The radius of the small auxiliary ⊙ is 6 in. − 1 in. = 5 in. ∴ length of common tangent $= \sqrt{13^2 - 5^2}$ in. = 12 in. *Ans.*

15. See Ex. of § 410. Hence, $12 : 18 = x : 20 - x$. $\therefore x = 8, 20 - x = 12$. *Ans.*

16. See the figure of § 319. Let $AB = 6$, $BC = 8$. Then $AC = \sqrt{6^2 + 8^2} = 10$. Also $AF : AB = AB : AC$. $\therefore AF : 6 = 6 : 10$. $\therefore AF = 3.6$. *Ans.* Similarly, $BC = 6.4$. *Ans.* Also $AF : BF = BF : FC$ or $3.6 : BF = BF : 6.4$. $\therefore BF = 4.8$. *Ans.*

PAGE 285

17.



Let ABC be the triangle and BD the altitude. Then $AB = 16$, $BC = 17$, $BD = 15$, $AD = \sqrt{16^2 - 15^2} = \sqrt{31}$. Also $DC = \sqrt{17^2 - 15^2} = 8$. $\therefore AC = 13.567+$. *Ans.*

18. Denote the segments of the hypotenuse by x and $10 - x$. Then $x : 4 = 4 : 10 - x$ (§ 319). $\therefore x = 2, 10 - x = 8$. The legs (by § 319) are $2\sqrt{5}$ or $4.472+$, and $4\sqrt{5}$ or $8.944+$. *Ans.*

19. $\frac{22 \times 28 \times 3000}{7 \times 5280 \times 12}$ mi. $= 4\frac{1}{6}$ mi. *Ans.*

20. $c = \frac{8800}{1400}$ yd. $d = \frac{c}{\pi} = \frac{8800 \times 7}{1400 \times 22}$ yd. $= 2$ yd. or 6 ft. *Ans.*

21. See § 385. 10.47 +; 14.49 +. *Ans.*

22. See § 385. Let x = no. of degrees in the arc. $\therefore 14 = \frac{x}{180°} \times \frac{22}{7} \times 6$. $\therefore x = 133\frac{7}{11}°$. *Ans.*

23. See § 385. 12 in. $= \frac{90°}{180°} \times \frac{22r}{7}$. $\therefore 2r = 15\frac{3}{11}$ in. *Ans.*

24. $2\pi R - 2\pi r = 132$ in. $- 88$ in. $\therefore \frac{44}{7}(R - r) = 44$ in. $\therefore R - r = 7$ in. *Ans.*

25. Velocity in miles per second $= \frac{44 \times 93{,}250{,}000}{7 \times 365\frac{1}{4} \times 24 \times 60 \times 60}$ mi. $= 18.5+$ mi.

26. The diagonal of the square $= \sqrt{5^2 + 5^2} = 5\sqrt{2}$. Hence, the radius of the circumscribed circle $= \frac{5\sqrt{2}}{2} = 3.535+$. Hence, $c = \frac{110\sqrt{2}}{7} = 22.223+$. *Ans.*

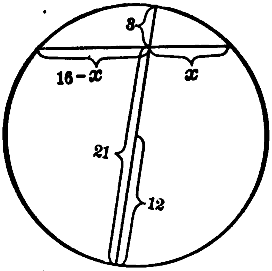
27. Diagonal of the rectangle $= \sqrt{12^2 + 5^2} = 13$. $\therefore r = \frac{13}{2}$. $c = 13\pi = 40.857+$. *Ans.*

28. Use § 385. Denote the central ∠ by x. Then $\frac{\pi r x}{180°} + 2r = \pi r$ or $\frac{\pi x}{180°} + 2 = \pi$. $\therefore x = 65\frac{5}{11}°$. *Ans.*

29. Denote a side of the square by x. $\therefore x^2 + x^2 = 100$ (§ 355). $\therefore x = 5\sqrt{2}$. $\therefore$ perimeter of square $= 20\sqrt{2}$. $\therefore 2\pi R = 20\sqrt{2}$. $\therefore R = 4.4997+$. *Ans.*

30. $x(14 - x) = 8 \times 3$ (§ 323). $\therefore x = 12$ or 2. *Ans.*

31.



$x(16 - x) = 21 \times 3$ (§ 323). $\therefore x = 9, 7$. $16 - x = 7, 9$. *Ans.*

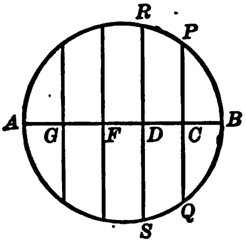
32. Denote the external segment of the second secant by x. Then $27x = 6 \times 24$ (§ 325). $\therefore x = 5\frac{1}{3}$. *Ans.*

33. The entire secant $= 9 + 16$ or 25. Let $x =$ length of the tangent. $\therefore 25 \times 9 = x^2$. $\therefore x = 15$. *Ans.*

PAGE 286

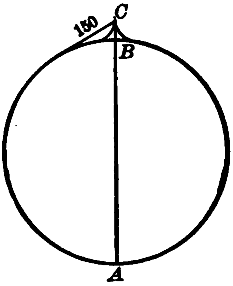
34. The line drawn from the point to the center ($= x$), the tangent, and the radius to the point of contact form a right △. $\therefore x^2 = \overline{24}^2 + \overline{18}^2$. $\therefore x = 30$. *Ans.*

35.



Let AB be the diameter and G, F, D, C the points of division. Then $AC = 48$, $CB = 12$. $\therefore PC = \sqrt{48 \times 12} = 24$ (§ 320), $PQ = 48$. Similarly, $RS = 24\sqrt{6}$ or 58.787 +, etc.

36.

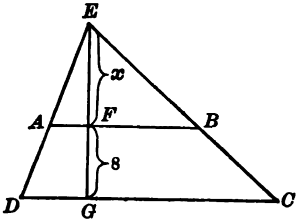


Let C be the top of the mountain. The limit of visibility is determined by the tangent drawn from the top of the mountain to the earth's surface. Denote the diameter of the earth by x. $\therefore AC = x + 3$. $\therefore 3(x + 3) = \overline{150}^2$ (§ 324). $\therefore x = 7497$ mi. *Ans.*

37. See Ex. 36. Denote the distance by x. But 100 ft. $= 0.019 +$ mi. $\therefore x^2 = 0.019\ (8000.019)$ mi. $\therefore x = 12.309 +$ mi. *Ans.*

38. The altitude of the upper △ is 10. Hence, to find the base of the upper △, 12 : 10 = 14: line drawn (§ 316). ∴ $11\frac{2}{3}$. *Ans.*

39.



Let *ABCD* be the trapezoid; let the legs produced meet in *E*, and *EG* be ⊥ *DC*. Denote *EF* by *x*. ▲ *AEB* and *DEC* are similar (§ 304). $x : x + 8 = 12 : 20$ (§ 316). $\therefore \left.\begin{array}{r} x = 12. \\ x + 8 = 20. \end{array}\right\}.$ *Ans.*

40. See solution of Ex. 39. $\frac{b_1 h}{b_2 - b_1}, \frac{b_2 h}{b_2 - b_1}$. *Ans.*

41. See formula 4 for h_c, p. 294. ∴ $\frac{21\sqrt{15}}{16}$ or 5.083 +. *Ans.*

42. See formula 5 for m_c, p. 294. ∴ $\frac{1}{2}\sqrt{106}$ or 5.147 +. *Ans.*

43. See formula 6 for t_c, p. 294. ∴ $\frac{21\sqrt{10}}{13}$ or 5.108 +. *Ans.*

44. See formulas for h_c, m_c, t_c, p. 294. The medians are $\frac{1}{2}\sqrt{673}$ or 12.971 +, $\frac{1}{2}\sqrt{592}$ or 12.165 +, $\frac{1}{2}\sqrt{505}$ or 11.236 +. The bisectors are $\frac{168\sqrt{5}}{29}$ or 12.953 +, $\frac{3}{2}\sqrt{65}$ or 12.093 +, $\frac{28\sqrt{13}}{9}$ or 11.217 +. The altitudes are $12\frac{12}{13}$ or 12.923 +, 12, 11.2. *Ans.*

1. No. acres = $\frac{300 \times 200}{2 \times 43560}$ = 0.688+. *Ans.*

2. Use Formula 5 (Areas), p. 294. 84. *Ans.*

3. Use Formula 5 (Areas), p. 294. 10 sq. ch. = 1 *A*. 203.33 + *A*. *Ans.*

4. Draw the altitude. Then alt. = $\sqrt{34^2 - 8^2}$, etc. 264.36+. *Ans.*

PAGE 287

5. Denote a side of the equilateral △ by $2x$. Then $4x^2 - x^2 = 64$ (§ 355). ∴ $2x = 9.236+$. Area = 36.9504+. *Ans.*

6. Denote a leg by x. $\therefore\ x^2 + x^2 = 144$ (§ 355). $\therefore\ x = 6\sqrt{2}$. Area $= \dfrac{6\sqrt{2} \times 6\sqrt{2}}{2} = 36$. *Ans.*

7. Other leg $\sqrt{41^2 - 9^2} = 40$ (§ 356). $\therefore$ area $= 180$. *Ans.*

8. $\dfrac{b^2\sqrt{3}}{4} = 4\sqrt{3}$. $\therefore\ b = 4$. *Ans.*

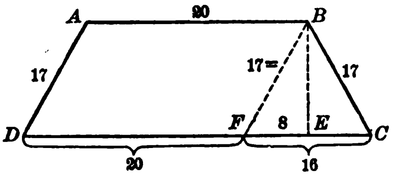
9. The no. of boards $= \dfrac{48 \times 24}{12 \times \frac{1}{2}} = 192$. *Ans.*

10. The no. of persons $= \dfrac{15 \times 9 \times 144}{27 \times 18} = 40$. *Ans.*

11. No. of acres $= \dfrac{90 \times 90}{43560} = 0.186-$. *Ans.*

12. See § 121. The diagonals divide the rhombus into four rt. $\triangle$, each of which has 17 for the hypotenuse and 15 for a leg. $\therefore\ \frac{1}{2}$ of the other diagonal $= \sqrt{17^2 - 15^2} = 8$. $\therefore$ area $= 240$. *Ans.*

13.



Let $ABCD$ be the trapezoid. Draw $BF \parallel AD$. $\therefore\ BFC$ is an isosceles $\triangle$ (§ 155). Draw its alt., BE. $\therefore\ BE = \sqrt{17^2 - 8^2} = 15$. $\therefore$ area $= 420$. *Ans.*

14. Denote the altitude of the upper $\triangle$ by x. $\therefore$ alt. of the trapezoid $= 18 - x$. Find, by use of similar $\triangle$, base of upper $\triangle = \dfrac{10x}{9}$. $\therefore$ area of trapezoid $= \left(\dfrac{5x}{9} + 10\right)(18 - x) = 80$. $\therefore\ x = 13.42-$. $\therefore$ upper base $= 14.907+$. *Ans.*

15. Draw lines from the center of the circle to the vertices of the polygon. In all the $\triangle$ thus formed the altitude is the same, viz.: the radius of the circle. $\therefore$ area of polygon $= \dfrac{20 \times 340}{2} = 3400$. *Ans.*

16. Denote the altitude by x. $\therefore\ (3x)\,x = 144$.

$$\left.\begin{aligned} \therefore\ x &= 4\sqrt{3}, \text{ or } 6.928+. \\ 3x &= 12\sqrt{3}, \text{ or } 20.785+. \end{aligned}\right\}\ \textit{Ans.}$$

17. Denote a side of the hexagon by x. Then $\frac{6x^2\sqrt{3}}{4} = 200$. $\therefore x =$ 8.77 + in. *Ans.*

18. By § 383, $c = 2\pi r$, or $p = 2\pi r$. $\therefore r = \frac{p}{2\pi}$. $\therefore K = \frac{\pi p^2}{4\pi^2} = \frac{p^2}{4\pi}$. (§ 390).

19. Denote the radius by x. $\therefore \pi x^2 = 81\pi + 1600\pi$. $\therefore x^2 = 1681$. $\therefore x = 41$ in. *Ans.*

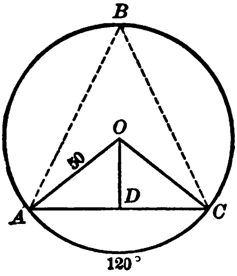
20. $\pi x^2 = \pi(\overline{20}^2 + \overline{28}^2 + \overline{29}^2)$. $\therefore x = 45$. *Ans.*

21. See § 394. $K = \frac{80°}{360°} \times \frac{22 \times \overline{50}^2}{7} = 1746+$. *Ans.*

22. Area of a sector of 60° $= \frac{60°}{360°} \times \frac{22 \times 2500}{7} = 1309.5238+$.

Area of an equilateral △ of side 50 $= \frac{\overline{50}^2\sqrt{3}}{4} = 1082.5319+$.

Area of a segment of 60° = 226.99 +. *Ans.* Area of a segment of 300° = (area of the circle) − (segment of 60°) = 7630.15 +. *Ans.* A segment of 240° is cut off by the side of an inscribed equilateral △ (segment AB). A side of △ ABC



$= 50\sqrt{3} = AC$. $\therefore OD = 25$. $\therefore$ segment $ABCD$ = sector $ABCO$ + △ AOC $= \frac{2}{3} \times \frac{22}{7} \times 2500 + \frac{1}{2} \times 50\sqrt{3} \times 25 = 6320.627+$. *Ans.*

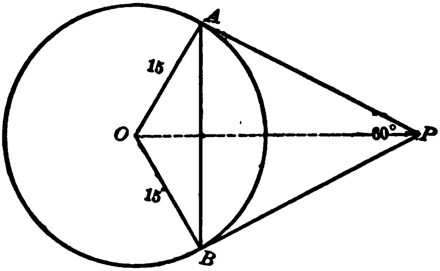
23. See § 394. Denote the angle by x. Then $\frac{x^2}{360°} \times \frac{22}{7} \times 49 =$ 45. $\therefore x = 105.2°$. *Ans.*

24. Side of inscribed square $= 10\sqrt{2}$. Sum of the segments = (area of whole circle) − (area of inscribed square) $= 100\pi - 200 =$ 114.28+. *Ans.*

PAGE 288

25. $\frac{1}{2}$ mi. = 880 yd. ∴ water area $= \frac{22}{4 \times 7}(\overline{880}^2 - \overline{100}^2) \times \frac{1}{4840}$ A. = 124.09 + A. *Ans.*

26.



Let O be the center of the circle and PA and PB tangents. △ ABP is isosceles ($PA = PB$, by § 216). ∴ $\angle PAB = 60° = \angle PBA$ (§ 102). ∴ $\angle OAB = 90° - 60° = 30° = \angle OBA$. ∴ $\angle AOB = 120°$. ∴ AB is a side of an inscribed equilateral △. ∴ $AB = 15\sqrt{3}$. ∴ area $OAPB = \triangle OAB + \triangle ABP = \frac{1}{2} \times 15 \times \frac{15\sqrt{3}}{2} + \frac{\overline{15}^2 \times 3\sqrt{3}}{4} = 225\sqrt{3} = 389.71+$. *Ans.*

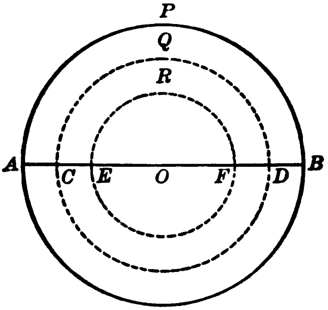
27. Denote the radius by x. ∴ $\frac{22}{7}x^2 = 43560$ sq. ft. ∴ $x = 117.7+$ ft. *Ans.*

28. Join the centers of the three circles. The lines of centers pass through the points of tangency (§ 221) and form an equilateral △. The included area is equal to an equilateral △ whose side is $2r$, minus the sum of three sectors which together equal $\frac{1}{2}$ of one of the circles. ∴ it $= \frac{(2r)^2\sqrt{3}}{4} - \frac{\pi r^2}{2} = r^2\left(\sqrt{3} - \frac{\pi}{2}\right)$.

29. The areas of the two similar ▲ are to each other as the squares of any pair of corresponding altitudes (§§ 352, 316, 291). Denote the area of the entire △ by K and of the upper by K'. ∴ $K : K' = \overline{18}^2 : 9^2 = 4 : 1$. But $K = 216$ sq. in. ∴ $K' = 54$ sq. in. *Ans.*

30. In this case K' must $= \frac{1}{2}K$. Denote the alt. of upper △ by x. ∴ $K : \frac{1}{2}K = \overline{18}^2 : x^2$. ∴ $x = 9\sqrt{2}$ in., or 12.728− in. *Ans.*

31. Denote area of circle *APB* by $3K$. Denote area of circle *ERF* by K. Denote area of circle *CQD* by $2K$. $3K : K = \overline{30}^2 : \overline{EF}^2$



(§ 393). $\therefore$ $EF = 10\sqrt{3}$ in. $= 17.320+$ in., also $3K : 2K = \overline{30}^2 : \overline{CD}^2$. $\therefore$ $CD = 10\sqrt{6}$ in. $= 24.495-$ in. *Ans.*

1. Alt. $= \sqrt{10^2 - 8^2} = 6$. $\therefore$ area $= 48$. *Ans.*

2. Area of $\triangle = 468$ by formula 5 (Areas), p. 294. Denote radius of the circle by x. Then $\pi x^2 = 468$. $\therefore$ $x = 12.228 +$. *Ans.*

3. $6\sqrt{3}$, or $10.392+$.

4. 14 in.; 16 in.; 19 in.; $7\sqrt{2}$ in.; $8\sqrt{2}$ in.; $9\sqrt{2}$ in. (or $9.899+$ in., $11.313 +$ in., $12.727+$ in.). *Ans.*

5. See Ex. of § 410. 9, 12. *Ans.*

6. By drawing the diagonal which does not cut the $\angle$ of 60°, the quadrilateral is divided into an equilateral and a rt. $\triangle$. $16.825+$. *Ans.*

7. Denote the diameter by x. Then $480 \times \pi x = 5280$ ft. $\therefore$ $x = 3.5$ ft. *Ans.*

PAGE 289

8. $\frac{h}{2}(12 + 16) = 112$ (§ 350). $\therefore$ $h = 8$. *Ans.*

9. Denote the radius of the circle by x. $\therefore$ $\pi x^2 = 100$ (§ 390). $\therefore$ $x = 5.6407+$. *Ans.*

10. $\pi x^2 = \frac{144\sqrt{3}}{4}$ (§ 390 and Ex. 4, p. 242). $\therefore$ $x = 4.4542-$. *Ans.*

11. $\pi x^2 = \frac{9}{2}(16 + 18)$ (§§ 390, 350). $\therefore$ $x = 6.977+$. *Ans.*

12. For area of circle see Ex. 18, p. 287. $K = \frac{\overline{12}^2}{4\pi}$ sq. yd. $= 11.45+$ sq. yd. *Ans.* Area of square $= 3^2$ sq. yd. $= 9$ sq. yd. *Ans.* Area of eq. $\triangle = \frac{4^2\sqrt{3}}{3}$ sq. yd. $= 6.9282+$ sq. yd. *Ans.*

13. $400 : 125 = 360° : \angle$ of sector. $\therefore$ $\angle$ of sector $= 112\frac{1}{2}°$. *Ans.*

14. Let $x =$ width of field included in ft. $6x =$ perimeter of running track in ft. $2x^2 =$ area included by track in sq. ft. But $6x = 2640$ ft. $\therefore$ $x = 440$ ft. $\therefore$ $2x^2 = \frac{440 \times 880}{43560}$ A. $= 8.8+$ A. *Ans.*

15. $r = \frac{6 \text{ in.}}{2} = 3$ in. $R = \sqrt{3^2 + 3^2}$ in. $= 3\sqrt{2}$ in.

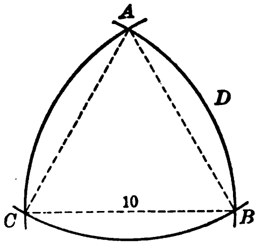
$$\text{By § 390, } \left.\begin{aligned} \pi r^2 &= 9\pi \text{ sq. in.} = 28.285 + \text{sq. in.} \\ \pi R^2 &= 18\pi \text{ sq. in.} = 56.571 + \text{sq. in.} \end{aligned}\right\} \textit{Ans.}$$

16. Denote the other leg by x. Then $x + 8 =$ hypotenuse. $\therefore$ $(x + 8)^2 = x^2 + \overline{12}^2$ (§ 355). $\therefore$ $x = 5$. $\therefore$ area $= 30$. *Ans.*

17. Let $x =$ a leg of the $\triangle$. $\therefore$ $x^2 + x^2 = \overline{20}^2$ (§ 355). $\therefore$ $x = 10\sqrt{2}$. Area $= \frac{1}{2}(10\sqrt{2})^2 = 100$. *Ans.*

18. Use § 319. $16, 4\sqrt{5}, 8\sqrt{5}$ (or 16, 8.944+, 17.888+). *Ans.*

19.



Let ABC be the given figure. Draw the chords of these arcs. These chords form an equilateral $\triangle$ whose side is 10 (§ 363). Area of $\triangle ABC = \frac{\overline{10}^2\sqrt{3}}{4} = 43.301+$. Area segment $ABD =$ sector $ACBD - \triangle ABC = \frac{\pi\overline{10}^2}{6} - 43.301 + = 9.07+$. $\therefore$ required area $= \triangle ABC + 3$ segment $ADB = 70.5403+$. *Ans.*

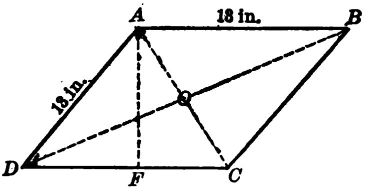
20. $\sqrt{43560} = 208.7 -$. $\therefore$ perimeter of square field $= 834.841+$ ft. For circle $43560 = \pi R^2$. $\therefore$ $R = \sqrt{\frac{43560}{\pi}}$. $\therefore$ $C = 2\pi\sqrt{\frac{43560}{\pi}} =$

$2\sqrt{\pi}\sqrt{43560}$ ft. = 740.016+ ft. $\therefore$ difference = (834.841 − 740.016 +) ft. = 94.83+ ft. *Ans.*

21. Area is doubled (§ 345); is doubled (§ 345); quadrupled (§ 346).

22. Denote the side of the $\triangle$ by x; then $\frac{x^2\sqrt{3}}{4} = \frac{100\pi}{4}$. $\therefore x =$ 13.47−. *Ans.*

23.



Let $ABCD$ be the given rhombus and AF its altitude. Then $AF \times 13$ in. = 156 sq. in. $\therefore AF = 12$ in. $\therefore DF = \sqrt{\overline{13}^2 - \overline{12}^2}$ in. = 5 in. $\therefore FC$ = 13 in. − 5 in. = 8 in. In the rt. $\triangle AFC$, $AC = \sqrt{\overline{12}^2 + 8^2}$ in. = $4\sqrt{13}$ in. = 14.422+ in. *Ans.* Area of rhombus = 156 sq. in. = $\frac{AC \times DB}{2} = 2\sqrt{13}$ in. $\times DB$. $\therefore DB$ $= \frac{156 \text{ in.}}{2\sqrt{13}} = 6\sqrt{13}$ in. = 21.633+ in. *Ans.*

PAGE 290

1. Alt. = 8 dm. $\therefore K$ = 64 sq. dm. (§ 343). *Ans.*

2. The sides are 6 m., 7 m., 8 m. $\therefore$ by Formula 5 (Areas), p. 294, $K = \frac{21}{4}\sqrt{15}$ sq. m. = 20.333+ sq. m. *Ans.*

3. R = 1.4 m. $\therefore K = \pi\overline{1.4}^2$ sq. m. = 6.16 + sq. m. *Ans.*

4. The other leg = $\sqrt{\overline{17}^2 - \overline{15}^2}$ dm. = 8 dm. $\therefore K$ = 60 sq. m. *Ans.*

5. See Ex. 18, p. 287. 7.9545 + sq. dm. *Ans.*

6. $K = \pi\overline{100}^2$ sq. m. = $\frac{10000}{10000}\pi$. Ha. = π. Ha. = 3.1428 + Ha. *Ans.* 3.14 Ha. = (3.14 +)(2.471 +)A. = 7.7659+ A. *Ans.*

7. The side = 8 dm. $\therefore$ other side = $\sqrt{\overline{35}^2 - 8^2}$ dm. = 34.07 + dm. Area = 2.7258 + sq. m. = 4225.09 + sq. in. *Ans.*

8. The bases are 6 m. and 2 m., and the alt. is 8 m. $\therefore K$ = 32 sq. m. *Ans.*

9. The dimensions are 70 m. and 200 m. $\therefore K$ = 1.4 Ha. = 3.459 + A. *Ans.*

10. 9 dm. and 4 dm. *Ans.*

11. Side of square = 18 in. $\therefore$ area of square = 324 sq. in. $\therefore \pi R^2 = 324$ sq. in. $\therefore R = \sqrt{\frac{324}{\pi}}$ in. $\therefore R$ in dm. $= \frac{10}{39.37}\sqrt{\frac{324}{\pi}}$ dm. $= 2.578 +$ dm. *Ans.*

12. Circf. of wheel in meters $= \frac{10000}{5000}$ m. = 2 m. $\therefore$ diameter of wheel $= \frac{2}{\pi}$ m. Diameter of wheel in feet $= \frac{2 \times 39.37}{\pi \times 12} = 2.087+$ ft. *Ans.*

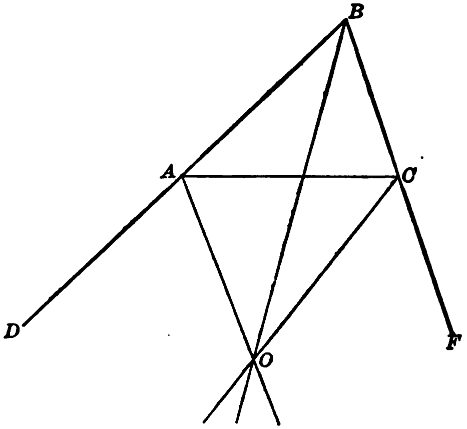
1. (Group 86). Use figure of Ex. 8, p. 97 (text-book). Denote the given adj. $\angle$s by $2x$ and $2y$. Then $x + y = 90°$ (Hyp.). $\therefore 2x + 2y = 180°$ (Ax. 4).

2. Use § 80.

PAGE 291

3. With the center of the given $\odot$ as a center and the given distance as a radius, describe a circle. From the given point draw a tangent to this circle. (§ 264), etc.

4.



Let $\angle ABO = \angle OBC$, and $\angle ACO = \angle OCF$. It is required to show that $\angle BOC = \frac{1}{2} \angle BAC$. Let $p = \angle ABO = \angle OBC$, $q = \angle BAC$, $r = \angle BCA$, $x = \angle BOC$. Then in $\triangle BOC$, $x + p + r + \frac{1}{2} \angle ACF = 180°$ (§ 102). Also in $\triangle ABC$, $2p + q + r = 180°$ (§ 102). But $\angle ACF = 2p + q$ (§ 103). $\therefore \frac{1}{2} \angle ACF = p + \frac{1}{2} q$ (Ax. 5). By Axs. 1 and 9, $x + p + r + p + \frac{1}{2} q = 2p + q + r$. $\therefore x = \frac{1}{2} q$ (Ax. 3).

5. Radius of inscribed $\odot = 9$ in., of circumsc. $\odot = 9\sqrt{2}$ in., etc.

6. Denote $\overset{\frown}{DB}$ by x, and $\overset{\frown}{BC}$ by y. Then $74° = \frac{1}{2}(x + 108°)$ (§ 240). $\therefore x = 40°$. Similarly, $106° = \frac{1}{2}(y + 112°)$. $\therefore y = 100°$. $\angle ADB = 104°$ (§ 235), etc.

7. A straight line which is the $\perp$ bisector of the line of centers. Use §§ 221, 123.

8. Use Ax. 5, §§ 82, 79, 23, 24.

9. Two pairs. See figure of § 322. (Draw chords DB and AC.) $\therefore$ $\triangle$ DFB and AFC are mutally equiangular (§ 235) and similar (§ 304). Likewise $\triangle$ DFA and BFC are similar.

10. Area of trapezoid = 44 sq. in. Area of square = 64 sq. in. To find per cent wasted, $\frac{60}{168} = .357+$. $\therefore$ 35.7 + per cent. *Ans.*

11. Draw AF. $\triangle ABF = \triangle AFE$ (§ 117). $\therefore BF = FE$. Then prove $FE = EC$, by showing $\angle ECF = 45° = \angle EFC$ (§§ 82, 102, 114).

12. Draw the radius to the point of contact. Thus a rt. $\triangle$ is formed (§ 210) whose hypotenuse is 24 cm. and one leg of which is 12 cm. $\sqrt{24^2 - 12^2}$ cm. $= 20.78+$ cm. *Ans.*

13. 36°. By § 168, each $\angle$ of a regular pentagon = 108°, etc.

14. Let the diagonals intersect at O. Then $OP = OR$ (Ax. 3). $OQ = OS$ (Ax. 3), etc.

15. See Ex. 7, p. 284. 3 in. *Ans.*

16. Produce PD to meet the circle at F. Then $\overset{\frown}{BF} = \overset{\frown}{BP}$ (§ 202). $\angle APB \overset{m}{=} \frac{1}{2}\overset{\frown}{BP}$ (§ 241), $\angle BPF \overset{m}{=} \overset{\frown}{BF}$ (§ 235), etc.

17. Reduces to § 255.

PAGE 292

18. Denote a leg of the given rt. $\triangle$ by x. Then $\frac{x^2}{2} = 1296$ sq. in. (§ 343). $\therefore x = 50.91 +$ in. *Ans.*

19. Let h = altitude of the given $\triangle$. Then h is a constant (§ 343). Hence, the locus is two lines $\parallel$ base of given $\triangle$ and at the distance h from this base.

20. In the $\triangle$ ACF and DCB, $DC = AC$ (sides of same square), $CF = CB$ (same reason), $\angle ACF = \angle BCD$ (each $= \angle ACB + 1$ rt. $\angle$), etc. Use §79.

21. Let $ABCD$ be the given quadrilateral and BP and PC the bisectors of the $\angle$ ABC, BCD. Denote each half of $\angle ABC$ by a, and each half of $\angle BCD$ by b. Then $2\angle a + 2\angle b + \angle D + \angle A = 360°$ (§ 167). Also $\angle a + \angle b + \angle P = 180°$ (§ 102). $\therefore 2\angle a + 2\angle b + \angle D + \angle A = 2\angle a + 2\angle b + 2\angle P$, etc.

22. Let AP and QB intersect in R outside the given circle.

$$\therefore \angle R \overset{m}{=} \tfrac{1}{2}(\overset{\frown}{AQ} - \overset{\frown}{BP}) \text{ (§ 242).}$$

$$\overset{m}{=} \tfrac{1}{2}(\overset{\frown}{AQ} + \overset{\frown}{AP} - \overset{\frown}{AP} - \overset{\frown}{BP}).$$

$$\overset{m}{=} \tfrac{1}{2}(180° - \overset{\frown}{AB}) = \text{a constant.}$$

Hence, if on the chord AB a segment of a circle is constructed which shall contain $\angle R$ (§ 265), the arc of the segment is the locus of R.

In like manner if AP and BQ intersect inside the circle it may be shown that $\angle ARB \overset{m}{=} \tfrac{1}{2}(180° + \overset{\frown}{AB})$.

23. Arc of $22\tfrac{1}{2}° = \dfrac{3\pi}{2}$; but $22\tfrac{1}{2}° = \tfrac{1}{16}(360°)$. $\therefore$ arc of $360° = 16\left(\dfrac{3\pi}{2}\right) = 24\pi$.

Hence, $2\pi r = 24\pi$. $\therefore r = 12$. $\therefore \pi r^2 = 452\tfrac{4}{7}$. Hence, $452\tfrac{4}{7}$ ft. *Ans.*

24. Denote the radii of the given circles by x and y. Then $\pi x^2 + \pi y^2 = 20$ sq. yd. $\pi x^2 - \pi y^2 = 15$ sq. yd., etc. $\therefore x = 2.359 +$ yd. $y = 0.891 +$ yd. *Ans.*

25. This problem reduces to one of constructing a right $\triangle$, given the hypotenuse (viz.: a side of the required square) and the sum of the legs (viz.: a side of the given square). See Ex. 14, p. 170.

26. Draw the diagonals of the ▱ and from their point of intersection draw a $\perp$ to same line. Then the sum of the $\perp$s from either pair of vertices of the ▱ = twice the $\perp$ from the point of intersection of the diagonals (Ex. 7, p. 213).

27. If the centers of the two circles are joined with each other and also with the points where the two circles intersect, two equilateral triangles are formed in each of which a side is r, the radius of each circle.

Hence, sum of areas of the two $\triangle$s $= \dfrac{r^2\sqrt{3}}{2}$ (Ex. 4, p. 242). The remainder of the area common to the two circles is four segments of 60° each in a circle with radius r. Area of one segment = sector of 60° − one of the $\triangle$s $= \dfrac{\pi r^2}{6} - \dfrac{r^2\sqrt{3}}{4} = \dfrac{2\pi r^2 - 3r^2\sqrt{3}}{12}$.

Hence, area common to the two circles

$$= \frac{r^2\sqrt{3}}{2} + \frac{4(2\pi r^2 - 3r^2\sqrt{3})}{12} = \frac{4\pi r^2 - 3r^2\sqrt{3}}{6}. \quad \textit{Ans.}$$

28. $\pi r^2 = 80. \quad \therefore r = \sqrt{\frac{280}{11}}.$

Arc of 80° $= (\frac{2}{9})\frac{44}{7}\sqrt{\frac{280}{11}} = \frac{8}{63}\sqrt{280 \times 11} = 7.04+.$ *Ans.*

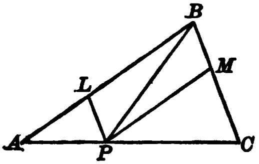
29. See Ex. 3, p. 241. Denote the radius of the inscribed circle by r.

$\therefore K = \frac{1}{2}(a + b + c)r.$

$$\therefore r = \frac{2K}{a+b+c} = \frac{2\sqrt{s(s-a)(s-b)(s-c)}}{a+b+c},$$

where $s = \frac{1}{2}(a + b + c)$. (See formula 5 under areas, p. 294.)

30.



Let $PLBM$ be the resulting ▱. $\therefore AL : LB = AP : PC = 2 : 3$ (§ 293). $\therefore \triangle ABP = \frac{2}{5} \triangle ABC$ (§ 345). $\therefore \triangle LBP = \frac{3}{5} \triangle ABP = \frac{6}{25} \triangle ABC$. ▱ $LBMP = 2 \triangle LBP = \frac{12}{25} \triangle ABC$. Hence, 12 : 25. *Ans.*

31. Let F be the midpoint of BE. From D draw $DH \parallel BE$ and meeting AB at H. Let K be the midpoint of HD. Then the locus is the broken line $CFKA$.

PAGE 293

32. Add $\triangle ECD$ to each of the equivalent $\triangle$s. $\therefore$ $\triangle$s BCD and ACD are equivalent. Since these $\triangle$s have the same base, CD, they must have equal altitudes, etc.

33. Let x be the exact diameter in inches of the required pipe. Then $2^2 : x^2 = 2 : 3. \quad \therefore x = \sqrt{6} = 2.76+$ (§ 393). $\therefore$ 3 in. pipe. *Ans.*